Cork Institute of Technology

CIT Mathematics Exam 2012

Sample Paper A

Paper 1

Time: 2 hours, 30 minutes

Total Marks for Paper 1 — 100 marks

Instructions

Question 1 is worth 40 marks.
Questions 2, 3, 4, 5, 6 are worth 20 marks each.

Answer **Question 1** and **any three** other questions.

Total marks available: 100 marks.

- The Formulae and Tables booklet (State Examinations Commission) is available at the examination.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.
- Answers should be given in simplest form, where relevant.

Q1

THIS QUESTION IS COMPULSORY.

Answer any **eight** parts [5 marks each].

(a) (i) A train leaves Cork at 07:50 and arrives in Dublin at 10:30. The distance from Cork to Dublin is 250 km. Find the average speed of the train in km/h.

(ii) John is booking accommodation in a hotel in Sydney. The rate per room per night in the hotel is 166 Australian dollars (AUD). The current exchange rate is €1 = AUD 0.69. What is the total price of five nights accommodation in the hotel? Give your answer in euro, correct to the nearest cent.

(b) (i) A box contains 400 electric light bulbs. 7.5% of them are faulty. Find the number of bulbs that are not faulty.

(ii) A prize fund is divided as follows:

the first prize is half of the fund,

the second prize is three-quarters of the first prize,

the third prize is what remains.

If the value of the third prize is €500, what is the total prize fund worth?

(c) Mary is competing in a 30 km walk.

(i) What fraction of the walk has she completed after the first 600 metres?

(ii) Express the fraction obtained in part (i) in scientific notation.

(iii) Mary completes the full walk in 3 hours 20 minutes. Express her average speed for the walk in metres per second.

(iv) Another competitor, Jessica, has an average speed for the walk of 2 metres per second. How long does it take her to complete the walk?

(d) Solve each of the following equations for x:

(i) $\dfrac{5x+6}{3} = x+4$

(ii) $x^2 - x - 6 = 0$

(iii) $\sqrt{x^2+7} - 1 = x$

[Q1 continued overleaf]

(e) (i) Express $\left(8^{\frac{1}{3}}\right)\left(4^{\frac{1}{4}}\right)$ in the form 2^k for $k \in \mathbb{Q}$.

(ii) Calculate the value of $(3\sqrt{5})^4$.

(iii) Calculate the value of

$$\frac{2.5 \times 10^3 + 1.2 \times 10^2}{7 \times 10^3}$$

and write your answer correct to 2 decimal places.

(f) Let $f(x) = \frac{2x}{3} - 4$, where $x \in \mathbb{R}$.

(i) Find the value of $f(6)$;

(ii) Find the value of x for which $f(x) = 14$;

(iii) If $g(x) = x^2$, find $g(f(0))$.

(g) Let $z = 2 + i$ where $i^2 = -1$. Plot each of the following numbers on an Argand diagram:

(i) $2z$

(ii) $iz + 1$

(iii) $\bar{z} + 5$ (where $\bar{z}$ denotes the complex conjugate of z)

(iv) z^2

(h) The letters of the word **DECIMAL** are arranged at random.

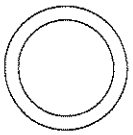
(i) In how many ways can this be done?

(ii) How many of these arrangements begin with the letter **D** and end with the letter **L**?

(iii) How many of these arrangements begin with a vowel and end with a consonant?

(iv) One of these arrangements is selected at random. What is the probability that it starts with a consonant?

(i) A circular flower bed has radius 7 metres. It is surrounded by a path of width 2 metres.



- Find the area of the path, correct to the nearest square metre.
- Hence calculate the cost of the path if paving costs €16.50 per square metre.

Q2

(a) Solve each of the following equations for x:

(i) $x^2 + 3x - 7 = 0$ (answer correct to one place of decimals)

(ii) $9^x = \frac{1}{3}$

[7 marks]

(b) Molly and Jim's ages add up to 30. Three times Molly's age is 5 more than twice Jim's age.
How old is Molly?

[4 marks]

(c) Solve for x and y:

$$\begin{aligned} 3x - 2y &= 17 \\ 2x + 7y &= 3 \end{aligned}$$

[4 marks]

(d) Solve for x and y:

$$\begin{aligned} x + y &= 1 \\ x^2 - y^2 &= 5 \end{aligned}$$

[5 marks]

Q3

(a) Let $f(x) = 2 - x^2 - x$ where $x \in \mathbb{R}$.

(i) Find the value of $f(0)$;

(ii) Solve the equation $f(x) = 0$;

(iii) Find $f'(x)$, the derivative of $f(x)$.

(iv) Hence find the coordinates of the local maximum point of the curve $y = f(x)$.

(v) Use your answers to parts (i) to (iv) of this question to sketch the graph of f in the domain $-3 \leq x \leq 2$.

[6 marks]

(b) Given that $y = \dfrac{2x}{x^2 + 3}$, find the value of $\dfrac{dy}{dx}$ when $x = 0$.

[4 marks]

(c) A car begins to slow down at a point P in order to stop at a red traffic light at a point Q.
The distance of the car (in metres) from P, after t seconds, is given by

$$s = 20t - \frac{1}{2}t^2$$

(i) Find the speed of the car five seconds after it passes P.

(ii) How long does it take the car to stop?

(iii) The car stops exactly at Q. Find the distance from P to Q.

[6 marks]

(d) Let $f(x) = (x^2 + 2)^3$, for $x \in \mathbb{R}$.
Find the equation of the tangent to the curve of $f(x)$ at the point where $x = -1$.

[4 marks]

Q4

(a) l is the line $5x - 2y + 5 = 0$.

(i) Find the slope of the line l.

(ii) Determine

- the point at which l intersects the x-axis;
- the point at which l intersects the y-axis.

(iii) Another line, k, passes through the points $(-4, 3)$ and $(6, 7)$. Find the equation of the line k.

(iv) Find the intersection point of the lines l and k.

(v) Draw a co-ordinate diagram on graph paper, showing the lines l and k and the point of intersection.

[10 marks]

(b) The circle c has equation $x^2 + y^2 = 25$.

(i) Write down the centre and the radius of the circle c.

(ii) Verify that the point $(2, 7)$ lies outside the circle c.

(iii) Write down the coordinates of any <u>two</u> points which lie <u>on</u> the circle c.

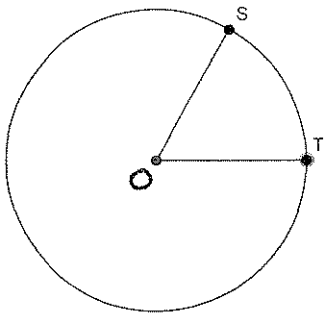
[5 marks]

(c) $A(1, 3)$, $B(3, 7)$ and $C(19, -1)$ are three points.
Show that the triangle ABC is right-angled at the point B.

[5 marks]

Q5

(a) A circle has centre O and radius 4 cm.
S and T are two points on the circle and $|\angle SOT| = 60°$.



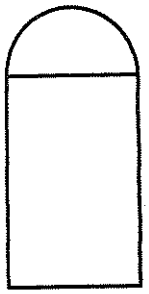
(i) Find the length of the shorter arc ST, in centimetres, correct to one decimal place.

(ii) Find the area of the sector SOT, in square centimetres, correct to two decimal places.

[5 marks]

(b) A window is in the shape of a rectangle surmounted by a semicircle, as shown below.
The rectangular part of the window is 140 cm high and 90 cm wide.
Find the area of the window, correct to the nearest square centimetre.

[4 marks]



[Q5 continued overleaf]

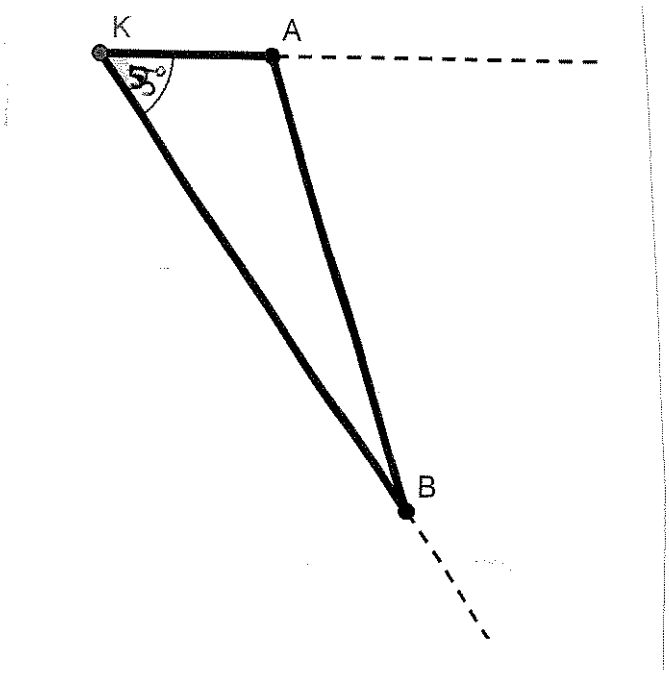
(c) The length of the shortest side of a right-angled triangle is 5 cm and the length of the hypotenuse is 13 cm.

(i) Find the length of the third side.

(ii) Find the value of the smallest angle of this triangle, correct to the nearest degree.

[5 marks]

(d) Two ships, A and B, leave port K at noon. Ship A is travelling due East and ship B is travelling East 55° South, as shown.



Calculate, to the nearest kilometre, the distance between ships A and B when A is 7 km from K and B is 11 km from K.

[6 marks]

Q6

(a) (i) How many different numbers, each with exactly 3 digits, can be formed from the digits 1, 2, 3, 4, 5? Each digit can be used only once in each number.

(ii) How many of these numbers are odd?

[4 marks]

(b) A bag contains 6 blue and 4 red discs. When a disc is drawn from the bag, it is returned before the next draw. What is the probability that two draws will yield

(i) both discs blue?

(ii) both discs the same colour?

[4 marks]

(c) Fifty students, girls and boys, donated blood recently. The blood type of each student was noted and the overall results are recorded in the following table:

Blood Type	A	B	O
Number of Girls	7	6	9
Number of Boys	9	6	13

If a student is selected at random, find the probability that

(i) the student is a girl with blood type A;

(ii) the student has blood type O;

(iii) the student is a boy;

(iv) the student has blood type A or O.

[5 marks]

(d) (i) The mean of the numbers 6, 5, x, 1, 11, 3 is 7.
Find the value of x.

(ii) The table below shows the number of goals scored in a recent soccer tournament:

Number of goals scored	0	1	2	3
Number of matches	3	12	17	k

The mean number of goals scored in a match was 2. Find the value of k.

[7 marks]

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Sample Paper B

Paper 1

Time: 2 hours, 30 minutes

Total Marks for Paper 1 — 100 marks

Instructions

Question 1 is worth 40 marks.
Questions 2, 3, 4, 5, 6 are worth 20 marks each.

Answer **Question 1** and **any three** other questions.

Total marks available: 100 marks.

- The Formulae and Tables booklet (State Examinations Commission) is available at the examination.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.
- Answers should be given in simplest form, where relevant.

Q1

THIS QUESTION IS COMPULSORY.

Answer any **eight** parts [5 marks each].

(a) (i) A photocopier broke down at 10:55. It was repaired at 12:40. For how many hours and minutes was the photocopier out of order?

(ii) Divide 800 grammes in the ratio 1 : 3 : 4.

(iii) A camera is priced at \$254.90 on a US website. Assuming an exchange rate of €1 = \$1·45, calculate the price in euro of the camera. Give your answer correct to the nearest cent.

(b) A car journey of 559 kilometres took 6 hours and 30 minutes.

(i) Calculate the average speed, in kilometres per hour, for the journey.

(ii) If the average diesel consumption for the journey was 21 kilometres per litre, calculate the amount of diesel used. Give your answer correct to the nearest litre.

(iii) If the cost of diesel is €1.44 per litre, calculate the cost of diesel for the journey.

(c) (i) The motorway speed limit in Ireland is 120 kilometres per hour. Express this speed in metres per second, correct to one decimal place.

(ii) A raffle is being held to raise money for a charity.

The cost of printing the raffle tickets is €80 for the first 500 tickets and €5 for each additional 100 tickets.

There are three prizes — the first prize is €200, the second prize is €100 and the third prize is €50. All three prizes are funded from the proceeds of the ticket sales.

If 1000 tickets are printed and 925 tickets are sold at €2 each, how much money will be raised for the charity after all costs have been met?

(d) (i) Solve the following equation for x:

$x(x+2)+16=4(2-x)$

(ii) Given that $t(4-a)=3(b+a)$, calculate the value of b when $a=-5$ and $t=2$.

(e) (i) Find the value of $\dfrac{5x-4y-2}{4}$ when $x=7$ and $y=3$.

(ii) Calculate the value of $(2\sqrt{2})^6$.

(iii) Solve the following equation for x: $3^{2x}=9$.

(f) Let $f(x)=x^3-x^2+2$.

(i) Verify that $f(3)-f(-3)=54$.

(ii) If $g(x)=2x$, find $g(f(0))$.

(g) Let $z=2+i$ where $i^2=-1$.
Plot each of the following numbers on an Argand diagram:

(i) z

(ii) iz

(iii) $\bar{z}+2z$ (where $\bar{z}$ denotes the complex conjugate of z)

(h) The letters in the word **VIOLA** are arranged taking all of the letters each time. How many different arrangements are possible if

(i) there are no restrictions?

(ii) the arrangements begin with the letter V?

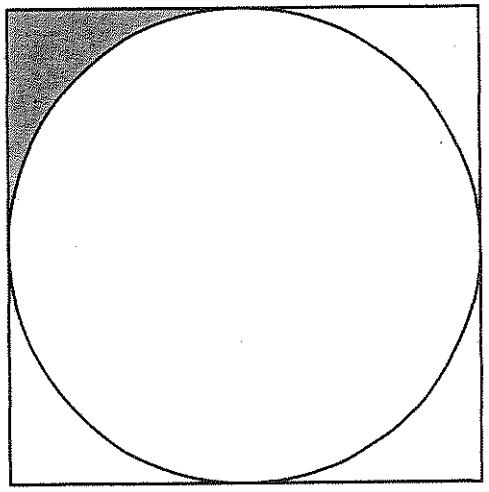
(iii) the arrangements begin with the letter V and end with a vowel?

(iv) the two consonants are together?

(i) A circle is inscribed in a square as shown.
The side length of the square is 10 cm.

- Find the area of the square.
- Find the area of the circle.
- Find the area of the shaded area.

Give all answers correct
to the nearest square centimetre.



Q2

(a) Solve each of the following equations for x:

(i) $5^{2x+1} = 125$

(ii) $\dfrac{2^x}{9} = \dfrac{1}{36}$

[5 marks]

(b) There are 100 seats in a cinema. Tickets cost €10 each for adults and €5 each for children. Last Friday afternoon, the cinema was full and the takings amounted to €660.

How many children were in the cinema?

[4 marks]

(c) Solve for x and y:

$$\begin{aligned} 3x - 2y &= 17 \\ 2x + 7y &= 3 \end{aligned}$$

[5 marks]

(d) (i) Write the following as a single fraction:

$$\frac{1}{x+1} + \frac{2}{x-3} \qquad x \neq -1 \text{ and } x \neq 3$$

(ii) Hence, or otherwise, find, correct to one place of decimals, the two solutions of the equation

$$\frac{1}{x+1} + \frac{2}{x-3} = 1, \qquad x \neq -1 \text{ and } x \neq 3$$

[6 marks]

Q3

(a) Let $f(x) = 2x^2 + 3x - 2$ where $x \in \mathbb{R}$.

(i) Find the value of $f(0)$;

(ii) Solve the equation $f(x) = 0$;

(iii) Find $f'(x)$, the derivative of $f(x)$.

(iv) Hence find the coordinates of the local minimum point of the curve $y = f(x)$.

(v) Use your answers to parts (i) to (iv) of this question to sketch the graph of f in the domain $-3 \leq x \leq 1$.

[8 marks]

(b) Given that $y = (x^3 - x^2 + 4)^3$, find the value of $\dfrac{dy}{dx}$ when $x = -1$.

[3 marks]

(c) Let $g(x) = x^3 + kx$.

(i) Given that $g(2) = 9$, show that $k = \dfrac{1}{2}$.

(ii) Hence find $g'(x)$.

(iii) Hence find the slope of the tangent to the curve of $g(x)$ at the point $(2, 9)$.

[4 marks]

(d) A firework was fired straight up into the air at $t = 0$ seconds.
The height h metres, which the firework reached above the ground t seconds after it was fired is given by

$$h = 80t - 5t^2$$

(i) At what height was the firework 2 seconds after it was fired?

(ii) The firework exploded 5 seconds after it was fired.
At what speed was it travelling when it exploded?

[5 marks]

Q4

(a) $A(-5, 9)$, $B(-1, 3)$, $C(1, -5)$ and $D(2, -4)$ are four points.

(i) Show all points on a co-ordinate diagram.

(ii) Find the slope of the line AC.

(iii) Show that the line AC is parallel to the line BD.

(iv) Find the equation of the line BD.

[6 marks]

(b) The circle c has equation $x^2 + y^2 = 100$.

(i) Write down the centre and the radius of the circle c.

(ii) Verify that the point $(5, 5)$ lies inside the circle c.

(iii) Write down the coordinates of the two points at which the circle c crosses the x-axis.

[5 marks]

(c) The line l has equation $x + 2y - 3 = 0$.

(i) Determine

- the point at which the line l intersects the x-axis;
- the point at which the line l intersects the y-axis.

(ii) Another line, k, has equation $x - y = 0$. Write down the co-ordinates of any two points on the line k.

(iii) Find the intersection point of the lines l and k.

(iv) Draw a co-ordinate diagram on graph paper, showing the lines l and k and the point of intersection.

[9 marks]

Q5

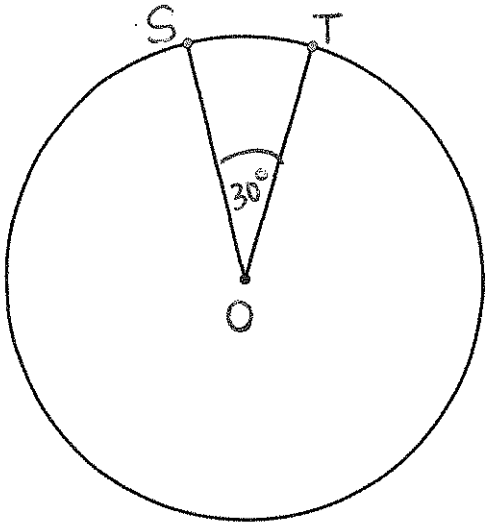
(a) A circle has centre O and radius 5 cm.

S and T are two points on the circle and $|\angle SOT| = 30°$.

Find, correct to two places of decimals:

(i) the area of the triangle SOT.

(ii) the area of the sector SOT.

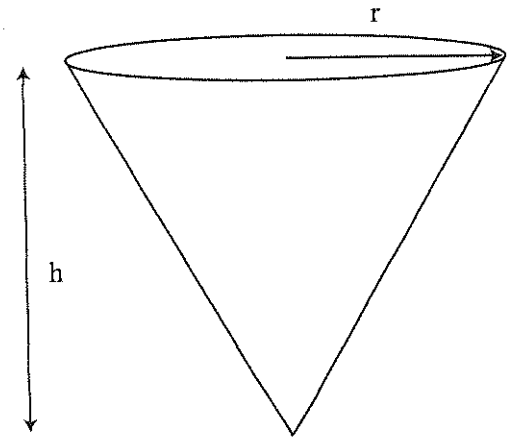


[4 marks]

(b) The conical tank shown has radius r metres and vertical height h metres. Its volume equals 18π m^3.

Given that $h = 2r$, find the radius and the vertical height of the tank.

[5 marks]



[Q5 continued overleaf]

(c) The length of the shortest side of a right-angled triangle is x cm while the lengths of the other two sides are 15 cm, 17 cm.

(i) Find the value of x.

(ii) Find the value of the smallest angle of this triangle, correct to the nearest degree.

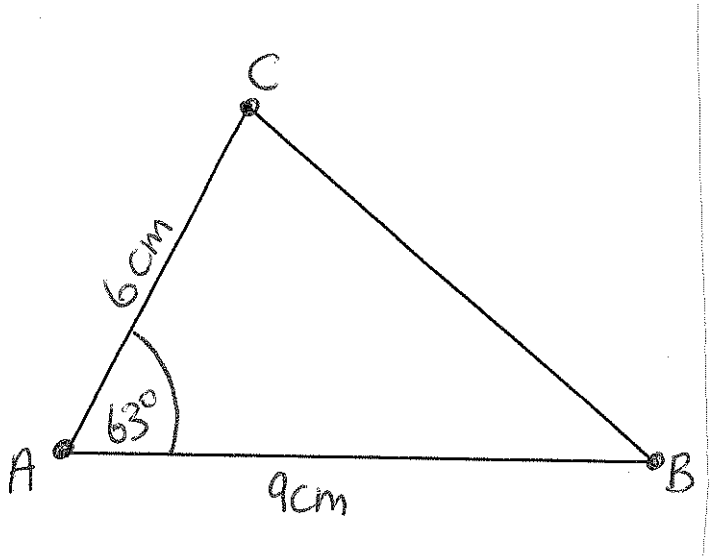
[4 marks]

(d) In the triangle ABC shown below,
$|AB| = 9$ cm, $|AC| = 6$ cm and $|\angle BAC| = 63°$.

(i) Find $|BC|$, correct to the nearest one-tenth of a centimetre.

(ii) Hence, or otherwise, find $|\angle ABC|$ and $\angle ACB|$, correct to the nearest degree.

[7 marks]



Q6

(a) Colm uses a password formed from one letter of his name followed by three of the digits from 0 to 9. He does not use any digit more than once.

(i) How many such passwords can be formed?

(ii) How many of the passwords begin with C?

(iii) How many of the passwords use only even digits?

[4 marks]

(b) I write down today's date **23082011** and then select one of the digits at random.

(i) What is the probability that I select a 2?

(ii) What is the probability that I select an odd digit?

[4 marks]

(c) A bag contains twenty coins: four 10-cent pieces, five 20-cent pieces, six 50-cent pieces and five euro coins.

A coin is selected at random from the bag.
Find the probability that

(i) the selected coin is not a euro coin.

(ii) the selected coin is worth 20 cent or less.

[4 marks]

(d) Six students are in a room. Their average height is 165 cm. When one student leaves the room, the average height of the remaining five students is reduced to 162 cm.
Find the height of the student who left the room.

[4 marks]

(e) The table below shows the number of occupants per car from a survey of fifty cars passing a fixed point:

Number of occupants	1	2	3	4	5
Number of cars	19	15	10	4	2

(i) How many persons were travelling in cars with 4 or more occupants?

(ii) Find the mean number of occupants per car.

[4 marks]