Cork Institute of Technology

CIT Mathematics Exam 2012

Sample Paper X

Paper 2

Time: 2 hours, 30 minutes

Total Marks for Paper 2 — 100 marks

Instructions

Answer **any five** of Questions 1, 2, 3, 4, 5, 6.
Each question is worth 20 marks.
Total marks available: 100 marks.

- The Formulae and Tables booklet (State Examinations Commission) is available at the examination.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.
- Answers should be given in simplest form, where relevant.

Q1

(a) Solve the following simultaneous equations for x, y and z:

$$\begin{aligned} x + y + z &= 5 \\ 2x - y + 3z &= 3 \\ 3x + 2y - 5z &= 21 \end{aligned}$$

[5 marks]

(b) (i) Given that $x = 1$ and $x = -2$ are roots of the cubic equation $mx^3 + nx^2 + x - 6 = 0$, find the value of the constants m and n.

Also find the third root of this equation.

(ii) Find all values of $k \in \mathbb{R}$ such that the equation

$$4x^2 + kx + 9 = 0$$

has two equal roots.

[6 marks]

(c) Solve each of the following equations for x:

(i) $x + \sqrt{x^2 + 1} = 3$

(ii) $\log_{10}(x + 4) = 2, \quad x \in \mathbb{R}, \quad x > -4$

[4 marks]

(d) Solve the inequality $|4x - 3| < 7$ for x.

[5 marks]

Q2

(a) An arithmetic sequence is such that the 4th term is 11 and the sum of the first 8 terms is 72.

(i) Find the first term and the common difference.

(ii) What term of the sequence is -13?

[4 marks]

(b) The 3rd term of a geometric sequence is 12. The 4th term of the sequence is -24.

(i) Find the first term and the common ratio of this sequence.

(ii) Hence find the sum of the first seven terms of this sequence.

[5 marks]

(c) Express the recurring decimal $1.434343\ldots$ in the form $\frac{p}{q}$ where $p, q \in \mathbb{N}$.

[5 marks]

(d) (i) Find the coefficient of $\frac{1}{x^2}$ in the binomial expansion of

$$\left(x^2 + \frac{1}{x}\right)^5$$

(ii) The coefficient of x^3 in the binomial expansion of $(2 + ax)^7$ is 4480. Find the value of a.

[6 marks]

Q3

(a) (i) Given that $|8 + bi| = 10$, find two possible values of the real constant b.

(ii) Given that

$$\frac{6 + ki}{4 - 3i} = 2i$$

find the value of the real number k.

[4 marks]

(b) (i) Express $z = -1 + i$ in polar form.

(ii) Deduce, using De Moivre's Theorem, that $z^6 = 8i$.

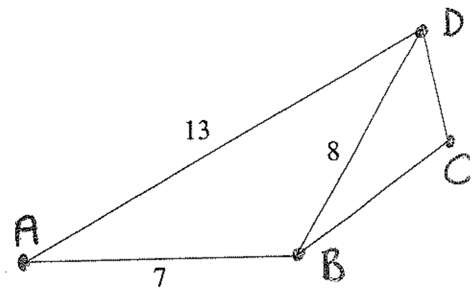
(iii) Hence find the value of z^{12}.

[7 marks]

(c) $ABCD$ is a quadrilateral in which $|AB| = 7$ cm, $|BD| = 8$ and $|AD| = 13$ cm.

(i) Show that $|\angle ABD| = 120°$.

(ii) Given that the area of the triangle BCD is $\frac{7\sqrt{3}}{2}$ m^2, find the area of the quadrilateral $ABCD$. Give your answer in surd form.



[5 marks]

(d) A is an acute angle such that

$$\sin\left(A + \frac{\pi}{3}\right) + \sin\left(A - \frac{\pi}{3}\right) = \frac{4}{5}$$

(i) Find $\sin A$.

(ii) Hence find $\cos A$.

[4 marks]

Q4

(a) $A = \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$.

(i) Find A^{-1}.

(ii) Hence find $A^{-1}(B+C)$.

[5 marks]

(b) A and B are points and O is the origin.
$\overrightarrow{a} = 4\overrightarrow{i} - 3\overrightarrow{j}$ and $\overrightarrow{b} = \overrightarrow{i} + 5\overrightarrow{j}$.

(i) Find a unit vector in the direction of $\overrightarrow{a}$.

(ii) C is a third point such that $\overrightarrow{AC} = \overrightarrow{OB}$.
Express $\overrightarrow{c}$ in terms of $\overrightarrow{i}$ and $\overrightarrow{j}$.

[5 marks]

(c) $P(5,-4)$, $Q(3,1)$ and $R(-5,-8)$ are three points and O is the origin.

Show that the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are perpendicular to each other.

[4 marks]

(d) (i) Find, in surd form, the value of $\cos A$, where A is the angle between vectors $\overrightarrow{u} = \overrightarrow{i} + 3\overrightarrow{j}$ and $\overrightarrow{v} = -2\overrightarrow{i} + 5\overrightarrow{j}$.

(ii) Find the value of A, correct to the nearest degree.

[6 marks]

Q5

(a) Find:

(i) the derivative of the function $y = x^2 e^{-x}$ with respect to x;

(ii) the value of the derivative of the function $y = \sin^2 x$ at $x = \frac{\pi}{4}$.

[5 marks]

(b) The parametric equations of a curve are:

$$x = t - \sin t \qquad \text{and} \qquad y = 1 - \cos t$$

(i) Find $\frac{dy}{dx}$ in terms of t.

(ii) Find the slope of the tangent to the curve at the point when $t = \frac{\pi}{2}$.

[5 marks]

(c) Given that $y = x^2 + \frac{16}{x^2}$, find $\frac{dy}{dx}$.

Hence verify that $y = x^2 + \frac{16}{x^2}$ has a minimum value at $x = 2$.

[6 marks]

(d) Find the equation of the tangent to the curve

$$x^2 - y^2 - x = 1$$

at the point $(2, 1)$.

[4 marks]

Q6

(a) Find

(i) $$\int 6x^2 + 4e^{-2x} + \frac{1}{\sqrt{x}}\, dx$$

(ii) $$\int_0^{\frac{\pi}{6}} \sin 4\theta \cos 2\theta \; d\theta$$

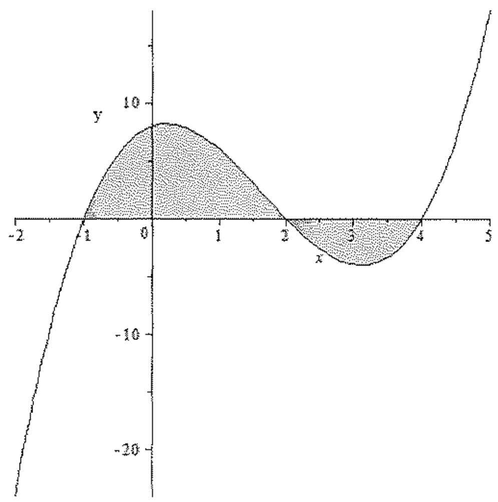
(iii) $$\int_0^1 \frac{x}{x^2+1}\, dx$$

[13 marks]

(b) The curve $y = x^3 - 5x^2 + 2x + 8$ crosses the x-axis at $x = -1$, $x = 2$ and $x = 4$, as shown.

Calculate the total area of the shaded regions enclosed by the curve and the x-axis.

[7 marks]



Cork Institute of Technology

CIT Mathematics Exam 2012

Sample Paper Y

Paper 2

Time: 2 hours, 30 minutes

Total Marks for Paper 2 — 100 marks

Instructions

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- Answers should be given in simplest form, where relevant.

Q1

(a) Solve the following simultaneous equations for x, y and z:

$$\begin{aligned} 3x - y + 3z &= 1 \\ x + 2y - 2z &= -1 \\ 4x - y + 5z &= 4 \end{aligned}$$

[5 marks]

(b) Given that $x = -2$ is a root of the cubic equation $x^3 + tx^2 + 3x - 10 = 0$, find the value of t.

Also find all other roots of this equation.

[4 marks]

(c) Solve each of the following equations for x:

(i) $\log_2(3x + 2) = 5, \quad x \in \mathbb{R}, \quad x > -\frac{2}{3}$

(ii) $\log_{10} 25 + 2\log_{10} 2 = x, \quad x \in \mathbb{R}, \quad x > 0$

[5 marks]

(d) Solve the inequality $\dfrac{x+1}{x-1} < 2$, where $x \in \mathbb{R}$ and $x \neq 1$.

Graph the solution set on a number line.

[6 marks]

Q2

(a) The first three terms of an arithmetic sequence are $2x+7$, $1-x$, $-3x-1$.

(i) Find the value of x.

(ii) Find the sum of the first twenty terms of the sequence.

[5 marks]

(b) The nth term of a sequence is given by $u_n = 3(2^n)$ for $n \in \mathbb{N}$.

(i) Prove that this sequence is geometric.

(ii) Write down the first three terms of this sequence.

(iii) Find the sum of the first 10 terms.

[4 marks]

(c) Express the recurring decimal $0.366666\ldots$ in the form $\frac{p}{q}$ where $p, q \in \mathbb{N}$.

[4 marks]

(d) (i) Find the value of the term which is independent of x in the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^6$.

(ii) The coefficient of x^4 in the binomial expansion of $(k+2x)^7$ is 15120. Find the value of k.

[7 marks]

Q3

(a) (i) Simplify $\frac{-2+3i}{3+2i}$.

Hence find the value of $\left(\frac{-2+3i}{3+2i}\right)^9$ where $i^2=-1$.

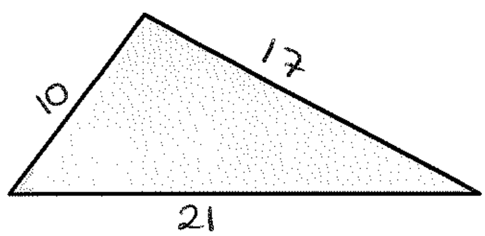
(ii) Express $z=\sqrt{3}+i$ in polar form. Hence evaluate z^{12}.

(iii) Find two complex numbers $a+ib$ such that

$$(a+ib)^2=-3-4i$$

[9 marks]

(b) The lengths of the sides of a triangle are 21, 17 and 10. The smallest angle in the triangle is A.



(i) Show that $\cos A=\frac{15}{17}$.

(ii) Without evaluating A, find $\cos 2A$.

[4 marks]

(c) Find the two solutions for θ of the equation

$$2\sin^2\theta+5\cos\theta+1=0, \text{ where } 0^\circ\le\theta\le180^\circ$$

Give your answers correct to the nearest degree.

[4 marks]

(d) The circumference of a circle is 30π cm. The area of a sector of this circle is 75π cm^2. Find, in radians, the measure of the angle in this sector.

[3 marks]

Q4

(a) $P = \begin{pmatrix} -4 & 2 \\ -7 & 3 \end{pmatrix}$ and $M = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$.

(i) Find P^{-1}.

(ii) Hence find $P^{-1}M$.

[5 marks]

(b) (i) Find the unit vector in the direction of $\vec{v} = 2\vec{i} - \vec{j}$.

(ii) If $\vec{w} = x\vec{i} + 5\vec{j}$ and $|\overrightarrow{2v + w}| = \sqrt{13}$, find two values for x.

[6 marks]

(c) P, Q and R are points and O is the origin.
$\vec{p} = 3\vec{i} + 4\vec{j}$, $\vec{q} = -8\vec{i} + 6\vec{j}$ and $\overrightarrow{PR} = 4\vec{i} + \vec{j}$.

(i) Express the vector $\overrightarrow{PQ}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Verify that the vectors $\vec{p}$ and $\vec{q}$ are perpendicular to each other.

(iv) Find θ, the angle between vectors $\vec{p}$ and $\vec{r}$. Give your answer correct to the nearest degree.

[9 marks]

Q5

(a) Find:

(i) the derivative of the function $y = \ln(\sin 4x)$ with respect to x;

(ii) the value of the derivative of the function $y = \sqrt{4x+1}$ at $x = 2$.

[5 marks]

(b) The parametric equations of a curve are:

$$x = 2t + t^2 \qquad \text{and} \qquad y = t + \frac{1}{t}, \qquad \text{where } t \in \mathbb{R}\backslash\{0\}$$

(i) Find $\frac{dy}{dx}$ in terms of t.

(ii) Find the equation of the tangent to the curve at the point given by $t = 2$.

[6 marks]

(c) Given that $y = x^2 e^{-2x}$, find $\frac{dy}{dx}$.
Hence verify that $y = x^2 e^{-2x}$ has a maximum value at $x = 1$.

[5 marks]

(d) Find the slope of the tangent to the circle

$$x^2 + y^2 + 2x - 3y - 13 = 0$$

at the point $(1, -2)$.

[4 marks]

Q6

(a) Find

(i) $$\int 2x^3 + 4\sin x \; dx$$

(ii) $$\int_1^4 \frac{1}{x^2} \; dx$$

[5 marks]

(b) Evaluate

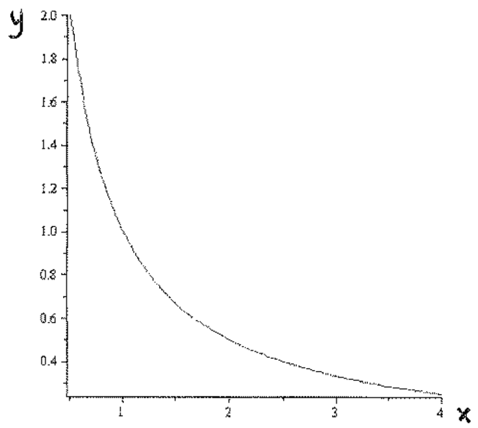
(i) $$\int_0^1 x\sqrt{1-x^2} \; dx$$

(ii) $$\int_0^{\frac{\pi}{6}} \sin 4x \sin 2x \; dx$$

(iii) $$\int_0^{\frac{3}{2}} \frac{1}{4x^2+9} \; dx$$

[11 marks]

(c) Calculate, correct to 2 places of decimals, the area of the bounded region enclosed by the line $x = 1$, the line $x = 4$ and the curve $y = \dfrac{1}{x}$



[4 marks]