

Cork Institute of Technology

Maths Qualifying Exam — SAMPLE PAPERS SET A

Paper 1 (100 marks)

(Time: 2.5 Hours)

Answer **QUESTION 1** and **THREE** other questions.

Question 1 is worth 40 marks.

Questions 2–5 are worth 20 marks each.

Total marks available: 100 marks.

- The standard **Mathematics Tables** booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

NB This question is compulsory.

Answer any **eight** parts [5 marks each].

- (a) (i) John works from 08:30 hours to 12:30 hours and again from 13:15 hours to 16:45 hours. He is paid €13.50 per hour.
Find his total pay for the day.
- (ii) A shop sells tennis rackets for €84 at a profit of 12%.
What is the cost price?
- (b) Tom is buying an mp3 player online. The sterling cost of the player is £63. If the exchange rate is €1 = £0.86 and the company charges £6.50 for post and packaging, what final amount must Tom pay in euro?
- (c) (i) A distance of 18 km is travelled in 10 minutes.
What is the average speed in metres per second?
- (ii) The sun is 1.5×10^{11} metres from the earth. Convert this distance to kilometres, giving your answer as a decimal number.
- (d) Write $\frac{4}{\sqrt{2}}$ as a power of 2.

Hence solve the following equation for x :

$$2^{4x+1} = \left(\frac{4}{\sqrt{2}}\right)^2$$

- (e) Express p in terms of q and t when

$$2p + q = 3(p - t)$$

Hence or otherwise, evaluate p when $q = 3^3$ and $t = 2^{-1}$.

- (f) Let $f(x) = 7 - 2x$, where $x \in \mathbb{R}$.
- (i) Find the value of $f(5)$;
- (ii) Find the value of x for which $f(x) = -1$;
- (iii) If $g(x) = f(x) - 10$, find the value of $g(5)$;
- (iv) If $h(x) = \frac{1}{x}$, find $h(f(5))$;

[Q1 continued overleaf]

- (g) Let $z = 3 - 2i$ where $i^2 = -1$. Plot each of the following numbers on an Argand diagram:
- (i) $z - 2$
 - (ii) $z + 3i$
 - (iii) $1 - iz$
- (h) Two people are chosen at random from a large crowd. Each person names the day of the week on which he or she was born. Assuming that each day is equally likely, what is the probability that
- (i) both people were born on a Monday?
 - (ii) one person was born on a Friday and the other person was born on a Saturday?
 - (iii) the two people were born on different days?
- (i) A cone has radius 5 cm and perpendicular height 12 cm. Find the slant height of the cone. Hence find the curved surface area of the cone, correct to the nearest square centimetre.

Q2

- (a) Solve each of the following equations:

(i) $x^2 - 4x + 1 = 0$

(ii) $(t + 1)^2 = 100$

[6 marks]

- (b) Solve for x and y

$$\begin{aligned} x + 2y &= 3 \\ x^2 - y^2 &= 24 \end{aligned}$$

[6 marks]

- (c) Tickets for a concert are priced as follows — adult tickets are priced at €10 each while child tickets are priced at €5 each. If 100 tickets were sold and the total takings for the concert were €850, how many adult tickets were sold?

[4 marks]

- (d) Solve for x the equation

$$\frac{x}{3} + \frac{1-x}{5} = 1$$

[4 marks]

Q3

- (a) Let $f(x) = 8 - 2x - x^2$, $x \in \mathbb{R}$.
- (i) Find the value of $f(0)$;
 - (ii) Solve the equation $f(x) = 0$;
 - (iii) Find $f'(x)$, the derivative of $f(x)$.
 - (iv) Hence find the coordinates of the local maximum point of the curve $y = f(x)$.
 - (v) Use your answers to parts (i)–(iv) of this question to sketch the graph of $f : x \rightarrow 8 - 2x - x^2$, showing scaled and labelled axes.

[8 marks]

- (b) Given that $y = (5x^2 + 3)(4 - x)$, find $\frac{dy}{dx}$ when $x = -1$.

[4 marks]

- (c) A marble rolls along the top of a table. It starts to move at $t = 0$ seconds. The distance s centimetres that it has travelled at t seconds is given by

$$s = 14t - t^2$$

- (i) What distance has the marble travelled when $t = 2$ seconds?
- (ii) When is the marble at rest?

[4 marks]

- (d) The slope of the tangent to the curve of $f(x) = 3x^2 + ax$ at the point where $x = -1$ is 3.

Find the value of a .

[4 marks]

Q4

- (a) $a(3, -1)$, $b(1, -4)$ and $c(5, 2)$ are three points.
- (i) Find the coordinates of the midpoint of the line segment $[ab]$.
 - (ii) Find $|ac|$, the distance between points a and c .
 - (iii) Find the equation of the line L which passes through b and c .
 - (iv) The point $(7, k)$ lies on the line L . Find the value of k .

[9 marks]

[Q4 continued overleaf]

(b) K_1 is the line $x - 2y + 1 = 0$ and K_2 is the line $3x + y - 11 = 0$.

(i) Show that the lines K_1 and K_2 are not parallel.

(ii) Find the point of intersection of the lines K_1 and K_2 .

[4 marks]

(c) The end points of a diameter of a circle are $(1, 5)$ and $(7, 3)$.

(i) Find the co-ordinates of the centre of this circle.

(ii) Find the radius length of this circle.

(iii) Find the equation of this circle.

(iv) Determine whether the point $(5, 4)$ lies inside, outside or on the circle.

[7 marks]

Q5

(a) A circle has centre o and radius 6 cm.

s and t are two points on the circle and $|\angle sot| = 60^\circ$.

(i) Find the length of the shorter arc st , correct to one decimal place.

(ii) Find the area of the sector sot , correct to the nearest square centimetre.

[5 marks]

(b) A solid metal ornament consists of a hemisphere of radius length 5 cm surmounted by a cone.

The slant height of the cone is four times the radius length of the hemisphere.

Find the volume of the ornament, correct to two decimal places.

[5 marks]

(c) Find, correct to two decimal places, the area of triangle abc if $|\angle abc| = 90^\circ$, $|ac| = 26$ cm and $|bc| = 10$ cm.

[5 marks]

(d) In a triangle xyz , $|\angle xzy| = 63^\circ$, $|xz| = 3$ cm and $|yz| = 4$ cm.

(i) Find $|xy|$, correct to two decimal places.

(ii) Find, correct to the nearest degree, the measure of the angle $\angle xyz$.

[5 marks]

Q6

- (a) A committee of 3 people is to be chosen from a group of 6 painters and 5 carpenters. In how many ways can the committee be selected
- (i) if there are no restrictions?
 - (ii) if the committee must contain exactly 2 carpenters?

[4 marks]

- (b) Liam uses a password formed from one letter of his name followed by four of the digits from 1 to 9. He does not use any digit more than once.
- (i) How many such passwords are possible?
 - (ii) How many of these passwords begin with a vowel?
 - (iii) How many of these passwords do not contain any odd digits?

[6 marks]

- (c) In a class, there are 17 boys and 15 girls. Four boys wear glasses and three girls wear glasses.

A pupil is chosen at random from the class. Find the probability that the pupil:

- (i) is a boy;
- (ii) wears glasses;
- (iii) is a boy who wears glasses;
- (iv) is a girl who does not wear glasses.

[6 marks]

- (d) The ages of children in a creche were as follows:

Age (years)	1	2	3	4	5
Frequency (number of children)	6	7	12	11	4

- (i) Find the total number of children in the creche.
- (ii) Find the mean age of children in the creche.

[4 marks]