

# Cork Institute of Technology

## Maths Qualifying Exam — SAMPLE PAPERS SET B

Paper 1 (100 marks)

(Time: 2.5 Hours)

Answer **QUESTION 1** and **THREE** other questions.

Question 1 is worth 40 marks.

Questions 2–5 are worth 20 marks each.

Total marks available: 100 marks.

- The standard **Mathematics Tables** booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

**Q1**

**NB** This question is compulsory.

Answer any **eight** parts [5 marks each].

- (a) (i) The scale on a map is 1:25000. The length of a wall on the map is 5 mm.  
Calculate, in metres, the actual length of the wall.

(ii) Write  $\frac{1.2 \times 10^3}{0.4 \times 10^2}$  as a decimal number.

- (b) Maria wants to bring £200 with her on holiday to London. If the exchange rate is €1 = £0.94 and the bank charges a transaction charge of €2.50, what will she have to pay the bank in euro?

- (c) Norma travels from her home to her workplace in Dublin, a distance of 72 km. The journey usually takes 1 hour 10 minutes.

- (i) Find her average speed in kilometres per hour for the journey.  
(ii) During school holidays, the traffic is lighter and Norma finds that her average speed for the journey increases to 80 km per hour.  
How long does Norma's journey to work take her on such days?

- (d) Solve each of the following equations for  $x$ :

(i)  $x(x + 4) - 3(2x + 1) = 0$

(ii)  $2^{3-x} = 4^x$

- (e) (i) Evaluate  $t[t(2t - 3) + 10] + 5$  when  $t = -0.5$ .

(ii) Express  $(6^4 \times 3^3) \div 12$  in the form  $2^a \times 3^b$ .

- (f) Let  $f(x) = 3x - 2$ , where  $x \in \mathbb{R}$ .

(i) Find the value of  $f(-1)$ ;

(ii) Find the value of  $x$  for which  $f(x) = 19$ ;

(iii) If  $g(x) = \frac{1}{x}$ , find  $g(f(-1))$  and  $f(g(-1))$ .

- (g) Given that  $i^2 = -1$ , find the value of

(i)  $i^4$

(ii)  $i^8$

(iii)  $i^3$

(iv)  $i^9$

[Q1 continued overleaf]

- (h) (i) Every PIN number for an ATM consists of four digits. The first digit is never zero. How many different PIN numbers are there?
- (ii) One letter is chosen at random from the letters of the word MEMORY.  
Find the probability that the letter chosen is a vowel.
- (i) Two cylinders have their radii in the ratio 2:1 and their heights in the ratio 1:3.  
What is the ratio of their volumes?

## Q2

- (a) Solve each of the following equations:

(i)  $2x^2 - 7x + 3 = 0$

(ii)  $2t = 3(5 - t)$

(iii)  $25^x = 5^{6-x}$

[6 marks]

- (b) Solve the following system of simultaneous equations for  $x$  and  $y$

$$3x - 2y = 5$$

$$2x + 3y = -1$$

[5 marks]

- (c) Tea served in a canteen is made from a mixture of two different types of tea, type A and type B. Type A costs €4 per kg. Type B costs €5 per kg. The mixture costs €4.60 per kg.

If the mixture contains 2 kg of type A, how many kilograms of type B does it contain?

[5 marks]

- (d) Express  $b$  in terms of  $a$  and  $c$  when  $\frac{8a - 5b}{b} = c$ .

[4 marks]

**Q3**

(a) Let  $f(x) = x^2 - 2x - 15$ ,  $x \in \mathbb{R}$ .

- (i) Find the value of  $f(0)$ ;
- (ii) Solve the equation  $f(x) = 0$ ;
- (iii) Find  $f'(x)$ , the derivative of  $f(x)$ .
- (iv) Hence find the coordinates of the local minimum point of the curve  $y = f(x)$ .
- (v) Use your answers to parts (i)–(iv) of this question to sketch the graph of  $f : x \rightarrow x^2 - 2x - 15$ , showing scaled and labelled axes.

[7 marks]

(b) Given that  $y = (t^2 + 1)^3$ , find  $\frac{dy}{dt}$  when  $t = 2$ .

[4 marks]

(c) A jet is moving along an airport runway. At the instant it passes a marker, it begins to accelerate for take-off. From the time the jet passes the marker, its distance from the marker is given by

$$s = 2t^2 + 3t$$

where  $s$  is in metres and  $t$  is in seconds.

- (i) Find the speed of the jet at the instance it passes the marker.
- (ii) The jet has to reach a speed of 83 metres per second to take off. After how many seconds will the jet reach that speed?

[4 marks]

(d) Let  $f(x) = \frac{1}{x-2}$ , for  $x \in \mathbb{R}$  and  $x \neq 2$ .

Find the equation of the tangent to the curve of  $f(x)$  at the point where  $x = 0$ .

[5 marks]

#### Q4

- (a)  $L$  is the line with equation  $4x - 3y - 6 = 0$ .
- (i) Find the slope of the line  $L$ .
  - (ii) Show that the point  $(3, 2)$  lies on  $L$ .
  - (iii) Determine the point at which  $L$  intercepts the  $x$ -axis.
  - (iv) Hence sketch a graph of the line  $L$ .
  - (v) Find the equation of the line which is perpendicular to  $L$  and which passes through the point  $(1, 3)$ .

[8 marks]

- (b)  $K_1$  is the line  $x - 2y + 1 = 0$  and  $K_2$  is the line which passes through the points  $(0, -11)$  and  $(4, 0)$ .
- (i) Show that the lines  $K_1$  and  $K_2$  are neither parallel nor perpendicular.
  - (ii) Find the equation of the line  $K_2$ .
  - (iii) Find the point of intersection of the lines  $K_1$  and  $K_2$ .

[6 marks]

- (c) The circle  $C$  has centre  $(0, 0)$  and passes through the point  $(8, 6)$
- (i) What is the radius of the circle  $C$ ?
  - (ii) Find the equation of the circle  $C$ .
  - (iii) Verify, by calculation, that the point  $(7, 7)$  lies inside the circle  $C$ .
  - (iv) Show the circle  $C$  on a co-ordinate diagram.  
Mark the four points at which the circle  $C$  intersects the axes and label them with their coordinates.

[6 marks]

**Q5**

- (a) A circle has centre  $o$  and radius 24 cm.

An arc of length 30 cm subtends an acute angle  $A$  at  $o$ .

Calculate  $A$ , correct to the nearest degree.

[4 marks]

- (b) A candle is in the shape of a cylinder surmounted by a cone.

The cone has perpendicular height 24 cm and the length of the radius of its base is 10 cm.

The height of the cylinder is equal to the slant height of the cone.

Find the volume of the candle, correct to two decimal places.

[5 marks]

- (c) Two ships,  $A$  and  $B$ , leave a port  $k$  at noon. Ship  $A$  is travelling due East while ship  $B$  is travelling due South.

Calculate, to the nearest km, the distance between the two ships when  $A$  is 6 km from  $k$  and  $B$  is 11 km from  $k$ .

[5 marks]

- (d) A triangular lot has sides which measure 100 m, 150 m and 300 m respectively.

(i) Find the measure of its largest angle.

(ii) Hence, or otherwise, find the measure of each of the other two angles.

[6 marks]

**Q6**

- (a) A fifth-year student has to choose three subjects from the following list:

Biology, Accounting, Technical Drawing, Art, Music, German.

(i) How many different choices are possible?

(ii) How many of these choices include Music?

[4 marks]

- (b) Four cards, numbered 2, 3, 4, 5 respectively, are shuffled and then placed in a row with the numbers visible.

Find the probability that the first and second numbers are even.

[4 marks]

[Q6 continued overleaf]

- (c) Twelve blood samples are tested in a laboratory. Of these, it is found that five blood samples are of type A, four are of type B and the remaining three are of type O.

Two blood samples are selected at random from the twelve.

What is the probability that

- (i) the two samples are of type A?
- (ii) one sample is of type B and the other sample is of type O?

[4 marks]

- (d) The following is the break-down of left-handers and right-handers in a tennis club, which has 200 members.

|        | Left – handed | Right – handed |
|--------|---------------|----------------|
| Male   | 24            | 67             |
| Female | 29            | 80             |

A member of the tennis club is chosen at random. Find the probability that the person is

- (i) a right-handed male;
- (ii) a female;
- (iii) left-handed.

[5 marks]

- (e) The mean of the following set of numbers is 4.5. Find the value of  $x$ .

3, 3, 6, 1,  $x$ , 5

[3 marks]