

Cork Institute of Technology

Maths Qualifying Exam — SAMPLE PAPERS SET A

Paper 2 (100 marks)

(Time: 2.5 Hours)

Answer **ANY FIVE** questions.

Each question is worth 20 marks.

Total marks available: 100 marks.

- The standard **Mathematics Tables** booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

- (a) Solve the following system of simultaneous equations:

$$\begin{aligned}x + 2y + 4z &= 7 \\x + 3y + 2z &= 1 \\-y + 3z &= 8\end{aligned}$$

[6 marks]

- (b) Show that
- $x = 2$
- is a root of the cubic equation
-
- $x^3 - 4x^2 + 9x - 10 = 0$
- .

Find all other roots of this equation.

[6 marks]

- (c) Solve the equation
- $x = \sqrt{x + 2}$
- for
- x
- .

[4 marks]

- (d) Solve for
- x
- :

$$|4x + 7| < 1$$

[4 marks]**Q2**

- (a) In an arithmetic series, the sum of the second term and the fifth term is 18.
-
- The sixth term is greater than the third term by 9.
-
- Find the first term and the common difference.

[4 marks]

- (b) The first three terms of a geometric sequence are

$$2x - 4, \quad x + 1, \quad x - 3.$$

Find the two possible values of x .**[4 marks]**

- (c) Express the recurring decimal
- $1.2222\dots$
- in the form
- $\frac{p}{q}$
-
- where
- $p, q \in \mathbb{N}$
- .

[6 marks]

- (d) Find the term independent of
- x
- in the binomial expansion of
- $\left(x^2 + \frac{1}{x}\right)^6$
- .

[6 marks]

Q3

- (a) (i) Given that $z = 2 - i$, calculate $|z^2 - z + 3|$ where $i^2 = -1$.
 (ii) Express $-1 + \sqrt{3}i$ in the form $r(\cos \theta + i \sin \theta)$, where $i^2 = -1$.
 (iii) Given that $z = 2 - 3i$, find the real number t such that $z^2 + tz$ is real.

[7 marks]

- (b) A is an acute angle such that $\tan A = \frac{3}{2}$.

Without evaluating A , find

- (i) $\cos A$
 (ii) $\cos 2A$

[4 marks]

- (c) Find all the solutions of the equation

$$\sin 2\theta + \sin \theta = 0$$

in the domain $0^\circ \leq \theta \leq 360^\circ$.**[5 marks]**

- (d) The diameter of a circle is 20 cm.
 The area of a sector of the circle is 75 cm^2 .
 Find, in radians and in degrees, the angle in this sector.

[4 marks]**Q4**

- (a) Let A, B be the matrices $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & -1 \\ -2 & 5 \end{bmatrix}$ respectively.

- (i) Find A^{-1} .
 (ii) Hence find the matrix C such that $AC = B$.

[5 marks]

- (b) $\vec{p} = 2\vec{i} + 3\vec{j}$ and \vec{p}^\perp is its related vector $-3\vec{i} + 2\vec{j}$.

Let $\vec{q} = \vec{p}^\perp - \vec{p}$ and $\vec{r} = \vec{p} + \vec{p}^\perp$.

- (i) Express \vec{q} and \vec{r} in terms of \vec{i} and \vec{j} .
 (ii) Find the measure of the angle between \vec{q} and \vec{r} .

[6 marks]**[Q4 continued overleaf]**

(c) $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = 3\vec{i} + k\vec{j}$, $\vec{c} = 4\vec{i} - 3\vec{j}$, where $k, t \in \mathbb{R}$ and o is the origin.

- (i) Find the unit vector in the direction of the vector \vec{c} .
- (ii) Find $|\vec{a}\vec{c}|$.
- (iii) Given that $\vec{a} \perp \vec{b}$, calculate the value of k .

[9 marks]

Q5

(a) Find the value of the derivative of the function $y = (3 - 4x)^5$ at $x = 1$.

[3 marks]

(b) The parametric equations of a curve are:

$$\begin{aligned}x &= \cos t + t \sin t \\y &= \sin t - t \cos t \quad \text{where } 0 < t < \frac{\pi}{2}\end{aligned}$$

Find $\frac{dy}{dx}$ and write your answer in its simplest form.

[6 marks]

(c) Let $f(x) = xe^{-3x}$, $x \in \mathbb{R}$.

- (i) Show that $f(x)$ has a local maximum and find the co-ordinates of the maximum point.
- (ii) Find the co-ordinates of the point at which the second derivative of $f(x)$ is zero.

[6 marks]

(d) Find the slope of the tangent to the curve

$$9x^2 + 4y^2 = 40 \quad \text{at the point } (2, 1)$$

[5 marks]

Q6

(a) Find:

(i)
$$\int 6x^3 + \frac{1}{x^2} + 4 \, dx$$

(ii)
$$\int e^{7x} \, dx$$

(iii)
$$\int \frac{x-2}{x^2-4x+5} \, dx$$

[6 marks]

(b) Find:

(i)
$$\int_0^3 \frac{12}{x^2+9} \, dx$$

(ii)
$$\int_0^{\pi/4} \sin 4x \sin 2x \, dx$$

[7 marks](c) Calculate the area bounded by the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$.**[7 marks]**