Cork Institute of Technology

Maths Qualifying Exam — SAMPLE PAPERS SET B

Paper 2 (100 marks)

(Time: 2.5 Hours)

Answer **ANY FIVE** questions.

Each question is worth 20 marks.
Total marks available: 100 marks.

- The standard Mathematics Tables booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

(a) Solve the following system of simultaneous equations:

$$\begin{aligned} 3x - y + 3z &= 1 \\ x + 2y - 2z &= -1 \\ 4x - y + 5z &= 4 \end{aligned}$$

[6 marks]

(b) Find the range of values of $x \in \mathbb{R}$ for which $x^2 + x - 12 \leq 0$.

[4 marks]

(c) Solve:

(i) the equation $\log_6(x + 5) = 2 + \log_6 x$ for $x > 0$

(ii) the equation $\log_9 x = \dfrac{3}{2}$ for $x > 0$

[6 marks]

(d) Solve for x : $|3x - 1| < 5$.

[4 marks]

Q2

(a) Three numbers are in arithmetic sequence.
Their sum is 21 and their product is 168.
Find the three numbers.

[4 marks]

(b) Find the sum to infinity of the following geometric series:

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$$

[4 marks]

(c) Express the recurring decimal $0.252525\ldots$ in the form $\dfrac{p}{q}$
where $p, q \in \mathbb{N}$.

[5 marks]

(d) Find the coefficient of x^2 in the binomial expansion of $\left(4x + \dfrac{1}{2x}\right)^8$.

[7 marks]

Q3

(a) (i) Given that $z = 2 - 4i$, calculate $|z + 2\bar{z}|$ where $i^2 = -1$ and $\bar{z}$ is the complex conjugate of z.

(ii) Express $5 - 5i$ in the form $r(\cos\theta + i\sin\theta)$, where $i^2 = -1$.

(iii) Given $w_1 = 4 + 3i$ and $w_2 = 6 - i$, express $\dfrac{w_1}{w_2}$ in the form $a + ib$ where $a, b \in \mathbb{R}$.

[7 marks]

(b) A is an acute angle such that $\tan A = \dfrac{8}{15}$.

Without evaluating A, find

(i) $\cos A$

(ii) $\sin 2A$

[4 marks]

(c) Find all values of θ for which

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

in the domain $0° \leq \theta \leq 360°$.

[5 marks]

(d) The circumference of a circle is 30π cm.
The area of a sector of the circle is 75 cm^2.
Find, in radians and in degrees, the angle in this sector.

[4 marks]

Q4

(a) Let A, B be the matrices $\begin{bmatrix} 8 & 2 \\ -1 & 4 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 \\ 0 & -5 \end{bmatrix}$ respectively.

Find $B^{-1}A$.

[4 marks]

[Q4 continued overleaf]

(b) (i) Find the unit vector in the direction of $\overrightarrow{r} = 3\overrightarrow{i} - 4\overrightarrow{j}$.

(ii) $oabc$ is a parallelogram where o is the origin, $\overrightarrow{a} = 3\overrightarrow{i} - \overrightarrow{j}$ and $\overrightarrow{b} = 4\overrightarrow{i} + 3\overrightarrow{j}$.
Express $\overrightarrow{c}$ in terms of $\overrightarrow{i}$ and $\overrightarrow{j}$.

[6 marks]

(c) rst is a triangle where $\overrightarrow{r} = -\overrightarrow{i} + 2\overrightarrow{j}$, $\overrightarrow{s} = -4\overrightarrow{i} - 2\overrightarrow{j}$ and $\overrightarrow{t} = 3\overrightarrow{i} - \overrightarrow{j}$.

(i) Express $\overrightarrow{rs}$, $\overrightarrow{st}$ and $\overrightarrow{tr}$ in terms of $\overrightarrow{i}$ and $\overrightarrow{j}$.

(ii) Show that the triangle rst is right-angled at r.

(iii) Find the measure of $\langle rst$.

[10 marks]

Q5

(a) Find:

(i) the derivative of the function $f(t) = \cos^4 t$ with respect to t;

(ii) the value of the derivative of the function $y = \sin^{-1} 10x$ at $x = \dfrac{1}{20}$.

[6 marks]

(b) The parametric equations of a curve are:

$$x = \ln(1 + t^2) \text{ and } y = \ln 2t \quad \text{where } t \in \mathbb{R}, t > 0$$

Find the value of $\dfrac{dy}{dx}$ at $x = \sqrt{5}$.

[5 marks]

(c) The function $f(x) = ax^3 + bx^2 + cx + d$ has a maximum point at $(0, 4)$ and a point of inflection at $(1, 0)$.
Find the values of a, b, c and d.

[5 marks]

(d) Find the slope of the tangent to the curve

$$9x^2 + 4y^2 = 40$$

at the point $(2, 1)$.

[4 marks]

Q6

(a) Find:

(i) $\int 1 + \frac{1}{x^3}\ dx$

(ii) $\int x^3 + \sqrt{x}\ dx$

[5 marks]

(b) Find:

(i) $\int_0^1 \frac{x}{x^2+4}\ dx$

(ii) $\int_0^{\frac{\pi}{2}} \sin^2 3\theta\ d\theta$

[7 marks]

(c) Calculate the area of the bounded region enclosed by the line $y = 2x - 1$, the line $x = 4$ and the curve $y = \frac{1}{x}$, where $x > 0$.

[8 marks]