

Cork Institute of Technology

CIT Mathematics Examination, 2015

Paper 1 — Sample Paper A

Answer **ALL FIVE** questions.

Each question is worth 20 marks.

Total marks available: 100 marks.

- The standard Formulae and Tables booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

- (a) (i) A camera is priced at \$254.90 on a US website. Assuming an exchange rate of 1 euro = \$1.45, calculate the price in euro of the camera. Give your answer correct to the nearest cent.
- (ii) A car journey of 559 kilometres took 6 hours and 30 minutes. Calculate the average speed, in kilometres per hour, for the journey.
- (iii) The motorway speed limit in Ireland is 120 kilometres per hour. Express this speed in metres per second, correct to one decimal place.
- (iv) Express 15 minutes as a percentage of 4 hours.

[7 marks]

- (b) Let $z_1 = 5 + i$ and $z_2 = 3 - 2i$, where $i^2 = -1$.
Plot each of the following numbers on an Argand diagram:

- (i) z_1
- (ii) z_2
- (iii) $z_1 - z_2$
- (iv) $z_1 z_2$

[6 marks]

- (c) The current population of Mathstown is 400,000. If the population grows at 2% per year, what will the population of Mathstown be in 5 years time?

[3 marks]

- (d) The first term of an arithmetic sequence is -1 and the third term of the same sequence is 3.

- (i) Find the common difference d , for this sequence.
- (ii) Hence find the sum of the first twelve terms of the sequence.

[4 marks]

Q2

- (a) Given that $t(4 - a) = 3(b + a)$, calculate the value of b when $a = -5$ and $t = 2$.

[3 marks]

- (b) Solve for x and y :

$$3x - 2y = 17$$

$$2x + 7y = 3$$

[4 marks]

- (c) (i) Write the following as a single fraction:

$$\frac{1}{x+1} + \frac{2}{x-3} \quad x \neq -1 \text{ and } x \neq 3$$

- (ii) Hence, or otherwise, find, correct to one place of decimals, the two solutions of the equation

$$\frac{1}{x+1} + \frac{2}{x-3} = 1, \quad x \neq -1 \text{ and } x \neq 3$$

[7 marks]

- (d) (i) Calculate the value of $(2\sqrt{2})^6$.
- (ii) Solve the following equation for x : $3^{2x} = 27$.
- (iii) Solve the following equation for x :

$$2^x = \frac{1}{16}$$

[6 marks]

Q3

(a) Colm uses a password formed from one letter of his name followed by three of the digits from 0 to 9. He does not use any digit more than once.

- (i) How many such passwords can be formed?
- (ii) How many of the passwords begin with C?
- (iii) How many of the passwords use only even digits?

[5 marks]

(b) I write down the date of this year's August Bank Holiday Monday **03082015** and then select one of the digits at random.

- (i) What is the probability that I select the 8?
- (ii) What is the probability that I select a zero?
- (iii) What is the probability that I select an odd digit?

[4 marks]

(c) A bag contains twenty coins: four 10-cent pieces, five 20-cent pieces, six 50-cent pieces and five euro coins. A coin is selected at random from the bag.

Find the probability that

- (i) the selected coin is a euro coin.
- (ii) the selected coin is not a euro coin.
- (iii) the selected coin is worth 20 cent or less.

[4 marks]

(d) Six students are in a room. Their average height is 165 cm. When one student leaves the room, the average height of the remaining five students is reduced to 162 cm.

Find the height of the student who left the room.

[3 marks]

(e) The table below shows the number of occupants per car from a survey of fifty cars passing a fixed point:

Number of occupants	1	2	3	4	5
Number of cars	19	15	10	4	2

- (i) How many persons were travelling in cars with 3 or more occupants?
- (ii) Find the mean number of occupants per car.

[4 marks]

Q4

(a) The circle c has equation $x^2 + y^2 = 100$.

- (i) Write down the centre and the radius of the circle c .
- (ii) Sketch a graph of the circle c on graph paper, marking
 - the points at which the circle c intersects the x -axis;
 - the points at which the circle c intersects the y -axis;

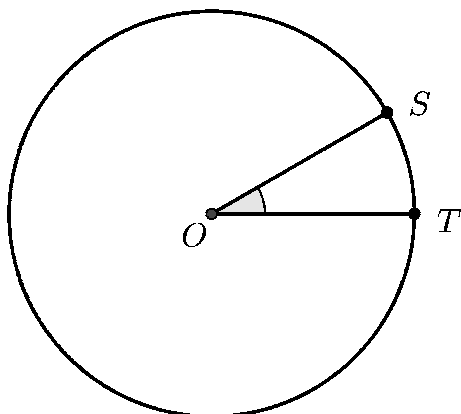
[3 marks]

(b) The circle shown below has centre O and radius 5 cm. S and T are two points on the circle and $|\angle SOT| = 30^\circ$.

Find, correct to two places of decimals:

- (i) the area of the triangle SOT .
- (ii) the area of the sector SOT .

[4 marks]



[Question 4 continues overleaf]

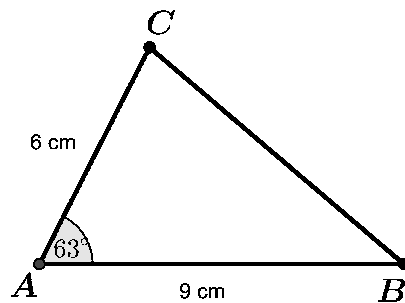
- (c) $A(-5, 9)$, $B(-1, 3)$, $C(1, -5)$ and $D(2, -4)$ are four points.
- Show all points on a co-ordinate diagram.
 - Find the slope of the line AC .
 - Show that the line AC is parallel to the line BD .
 - Find the equation of the line BD .

[6 marks]

- (d) In the triangle ABC shown below,
 $|AB| = 9$ cm, $|AC| = 6$ cm and $|\angle BAC| = 63^\circ$.

- Find $|BC|$, correct to the nearest one-tenth of a centimetre.
- Hence, or otherwise, find $|\angle ABC|$ and $|\angle ACB|$, correct to the nearest degree.

[7 marks]



Q5

(a) Let $f(x) = 2x^2 + 3x - 2$ where $x \in \mathbb{R}$.

- (i) Find the value of $f(0)$;
- (ii) Solve the equation $f(x) = 0$;
- (iii) Find $f'(x)$, the derivative of $f(x)$.
- (iv) Hence find the coordinates of the local minimum point of the curve $y = f(x)$.
- (v) Use your answers to parts (i) to (iv) of this question to sketch the graph of f in the domain $-3 \leq x \leq 1$.

[7 marks]

(b) Given that $y = x^3 - x^2 + 4$, find the value of $\frac{dy}{dx}$ when $x = -1$.

[3 marks]

(c) Let $g(x) = x^3 + kx$.

- (i) Given that $g(2) = 9$, show that $k = \frac{1}{2}$.
- (ii) Hence find $g'(x)$.
- (iii) Hence find the slope of the tangent to the curve of $g(x)$ at the point $(2, 9)$.

[5 marks]

(d) A firework was fired straight up into the air at $t = 0$ seconds. The height h metres, which the firework reached above the ground t seconds after it was fired is given by

$$h = 80t - 5t^2$$

- (i) At what height was the firework 2 seconds after it was fired?
- (ii) The firework exploded 5 seconds after it was fired. At what speed was it travelling when it exploded?

[5 marks]

Cork Institute of Technology

CIT Mathematics Examination, 2015

Paper 1 — Sample Paper B

Answer **ALL FIVE** questions.

Each question is worth 20 marks.

Total marks available: 100 marks.

- The standard Formulae and Tables booklet is available.
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- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

- (a) A prize fund is divided as follows:

the first prize is half of the fund,
the second prize is three-quarters of the first prize,
the third prize is what remains.

If the value of the third prize is 500 euro, what is the total prize fund worth?

[4 marks]

- (b) Mary is competing in a 30 km walk.

- (i) What fraction of the walk has she completed after the first 600 metres?
- (ii) Mary completes the full walk in 3 hours 20 minutes. Express her average speed for the walk in metres per second.
- (iii) Another competitor, Jessica, has an average speed for the walk of 2 metres per second. How long does it take her to complete the walk?

[6 marks]

- (c) Let $z = 2 + i$ where $i^2 = -1$. Plot each of the following numbers on an Argand diagram:

- (i) $-z$
- (ii) \bar{z} (where \bar{z} denotes the complex conjugate of z)
- (iii) iz
- (iv) $1 + iz + i$

[7 marks]

- (d) The mass of a single atom of aluminium is 4.4805×10^{-23} grams. Find, in kilograms, the mass of 100 million such atoms. Give your answer in scientific notation.

[3 marks]

Q2

(a) Solve the equation $x^2 + 2x - 6 = 0$ for x . Express the solutions in each of the following forms:

- in the form $a + \sqrt{b}$, where $a, b \in \mathbb{Z}$.
- in decimal form, correct to 4 significant figures.

[5 marks]

(b) (i) Express $\left(8^{\frac{1}{3}}\right)\left(4^{\frac{1}{4}}\right)$ in the form 2^k for $k \in \mathbb{Q}$.

(ii) Calculate the value of

$$\frac{2.5 \times 10^3 + 1.2 \times 10^2}{7 \times 10^3}$$

and write your answer correct to 2 decimal places.

[7 marks]

(c) Solve the following equations for x :

(i) $9^x = \frac{1}{3}$

(ii) $10^{2x} = 10000$

[4 marks]

(d) Molly and Jim's ages add up to 30. Three times Molly's age is 5 more than twice Jim's age.

How old is Molly?

[4 marks]

Q3

- (a) (i) How many different numbers, each with exactly 3 digits, can be formed from the digits 1, 2, 3, 4, 5? Each digit can be used only once in each number.
- (ii) How many of these numbers are odd?

[3 marks]

- (b) A bag contains 6 blue and 4 red discs. When a disc is drawn from the bag, it is returned before the next draw. What is the probability that two draws will yield

- (i) both discs blue?
- (ii) both discs the same colour?

[4 marks]

- (c) Fifty students, girls and boys, donated blood recently. The blood type of each student was noted and the overall results are recorded in the following table:

Blood Type	A	B	O
Number of Girls	7	6	9
Number of Boys	9	6	13

If a student is selected at random, find the probability that

- (i) the student is a girl with blood type A;
- (ii) the student has blood type O;
- (iii) the student does not have blood type O;
- (iv) the student is a boy;
- (v) the student has blood type A or O.

[6 marks]

- (d) The following is a list of the results of a recent CIT Maths exam, taken by 20 students. Marks are out of 10.

3	4	4	4	5
5	5	5	7	7
7	7	7	7	8
8	8	9	9	10

- (i) Find the median mark.
- (ii) Find the mean mark for the class.

[7 marks]

Q4

(a) l is the line $5x - 2y + 5 = 0$.

- (i) Find the slope of the line l .
- (ii) Determine
 - the point at which l intersects the x -axis;
 - the point at which l intersects the y -axis.
- (iii) Another line, k , passes through the points $(-4, 3)$ and $(6, 7)$. Find the equation of the line k .
- (iv) Find the intersection point of the lines l and k .
- (v) Draw a co-ordinate diagram on graph paper, showing the lines l and k and the point of intersection.

[10 marks]

(b) The length of the shortest side of a right-angled triangle is 5 cm and the length of the hypotenuse is 13 cm.

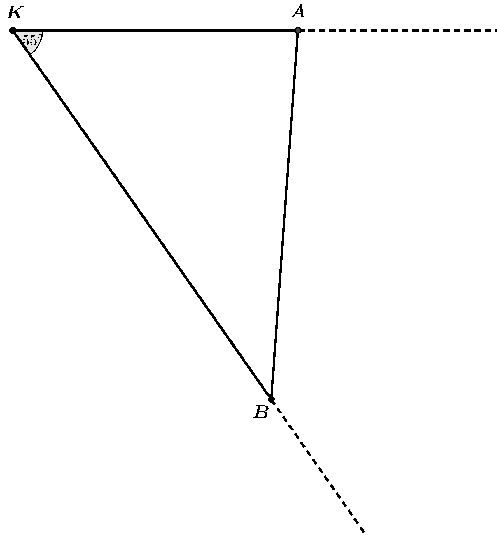
- (i) Find the length of the third side.
- (ii) Find the value of the smallest angle of this triangle, correct to the nearest degree.

[5 marks]

[Q4 continued overleaf]

- (c) Two ships, A and B, leave port K at noon. Ship A is travelling due East and ship B is travelling East 55° South, as shown.
Calculate, to the nearest kilometre, the distance between ships A and B when A is 7 km from K and B is 11 km from K .

[5 marks]



Q5

(a) Let $f(x) = 2 - x^2 - x$ where $x \in \mathbb{R}$.

- (i) Find the value of $f(0)$;
- (ii) Solve the equation $f(x) = 0$;
- (iii) Find $f'(x)$, the derivative of $f(x)$.
- (iv) Hence find the coordinates of the local maximum point of the curve $y = f(x)$.
- (v) Use your answers to parts (i) to (iv) of this question to sketch the graph of f in the domain $-3 \leq x \leq 2$.

[6 marks]

(b) Given that $y = 3x^2 - 2x + 5$, find the value of $\frac{dy}{dx}$ when $x = -2$.

[4 marks]

(c) A car begins to slow down at a point P in order to stop at a red traffic light at a point Q .

The distance of the car (in metres) from P , after t seconds, is given by

$$s = 20t - \frac{1}{2}t^2$$

- (i) Find the speed of the car five seconds after it passes P .
- (ii) How long does it take the car to stop?
- (iii) The car stops exactly at Q . Find the distance from P to Q .

[6 marks]

(d) Let $f(x) = 4 - 3x - x^2$, for $x \in \mathbb{R}$.

Find the equation of the tangent to the curve of $f(x)$ at the point where $x = -1$.

[4 marks]