Cork Institute of Technology

CIT Mathematics Examination, 2015

Paper 2 — Sample Paper A

Answer **ALL FIVE** questions.

Each question is worth 20 marks.
Total marks available: 100 marks.

- The standard Formulae and Tables booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

(a) (i) Given that $|8 + bi| = 10$, find two possible values of the real constant b.

(ii) Given that

$$\frac{6 + ki}{4 - 3i} = 2i$$

find the value of the real number k.

[6 marks]

(b) (i) Express $z = -1 + i$ in polar form.

(ii) Deduce, using De Moivre's Theorem, that $z^6 = 8i$.

(iii) Hence find the value of z^{12}.

[6 marks]

(c) In a geometric series, the sum of the 1st and 3rd terms is 250. The sum of the second and fourth terms is -125.

(i) Find the first term and the common ratio of this series.

(ii) Show that S_∞, the sum to infinity of the series, exists. Calculate the value of S_∞.

[4 marks]

(d) Express the recurring decimal $1.434343\ldots$ in the form $\dfrac{p}{q}$ where $p, q \in \mathbb{N}$.

[4 marks]

Q2

(a) Solve each of the following equations for x:

(i) $x + \sqrt{x^2 + 1} = 3$

(ii) $\log_{10}(x + 4) = 2, \quad x \in \mathbb{R}, \quad x > -4$

[5 marks]

(b) Solve the following simultaneous equations for x, y and z:

$$\begin{aligned} x + y + z &= 5 \\ 2x - y + 3z &= 3 \\ 3x + 2y - 5z &= 21 \end{aligned}$$

[5 marks]

(c) Given that $x = 1$ and $x = -2$ are roots of the cubic equation $mx^3 + nx^2 + x - 6 = 0$, find the value of the constants m and n.

Also find the third root of this equation.

[5 marks]

(d) A company is trying to expose as many people as possible to a new product through television advertising in a large city with 4 million possible viewers. A model for the number of people N, in millions, who are aware of the product after t days of advertising was found to be

$$N = 4(1 - e^{-0.037t})$$

(i) How many people are aware of the product after the first two days of the campaign?

(ii) How many days, to the nearest day, will the advertising campaign have to last so that 50% of the possible viewers will be aware of the product?

[5 marks]

Q3

(a) A fair dice is rolled once. Let A be the event that an odd number is obtained, let B be the event that a 2 is obtained, and let C be the event that the number obtained is strictly greater than 4.
For each of the following statements, please indicate if that statement is true or false. Justify your answer in each case.

(i) Events A and B are mutually exclusive.

(ii) Events A and C are mutually exclusive.

(iii) Events B and C are mutually exclusive.

[5 marks]

(b) Of a class of 30 students, 12 students are studying French while 15 are studying German. It is also known that exactly two students are studying both languages. Calculate the probability that a randomly selected student from the class

(a) is studying both languages;
(b) is studying at least one of these two languages;
(c) is studying neither French nor German;
(d) is studying exactly one of these two languages.

[5 marks]

(c) A telephone based automated customer care system has two main menu options, Option 1 and Option 2.
In general, 80% of customers who select Option 1 are eventually routed to a service agent, while 90% of customers who select Option 2 are eventually routed to a service agent.
Yesterday, 2000 customers contacted the customer care system, with 60% of these selecting Option 1 and the remaining 40% selecting Option 2.

(i) Determine the number of customers who selected Option 1 and who were routed to a service agent.

(ii) Find the probability that a customer who selected Option 2 was routed to a service agent.

(iii) Find the probability that a randomly selected customer was routed to a service agent.

[5 marks]

Question 3 continues overleaf

(d) An auditor claims that 10% of a company's invoices are incorrect. To test this claim, a random sample of 200 invoices was checked, and 28 were found to be incorrect. Test the auditor's claim, at the 5% level of significance.

[5 marks]

Q4

(a) The centre of a circle lies on the line $x - 2y - 1 = 0$. The $x-$axis and the line $y = 6$ are tangents to the circle. Find the equation of this circle.

[5 marks]

(b) Find all the solutions to the equation

$$\sin 2x = \frac{\sqrt{3}}{2}, \qquad \text{for } 0^\circ \leq x \leq 360^\circ$$

[4 marks]

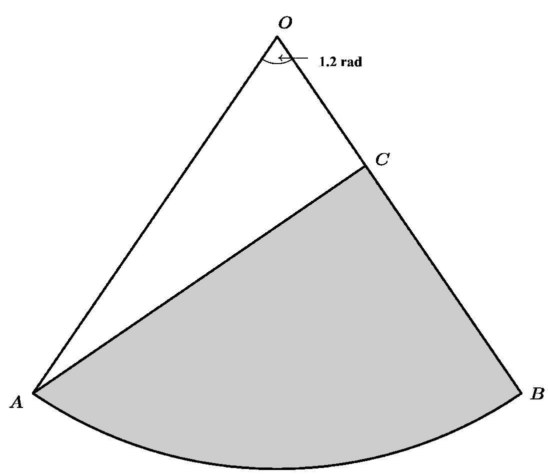
(c) OAB is a sector of a circle, centre O, radius 10 cm and $|\angle AOB| = 1.2$ radians, as shown in the diagram below.
Given that $AC \perp OB$:

(i) Show that $|BC| = 6.38$ cm.

(ii) Calculate the perimeter of the shaded region.

(iii) Calculate the area of the shaded region.

[6 marks]



Question 4 continues overleaf

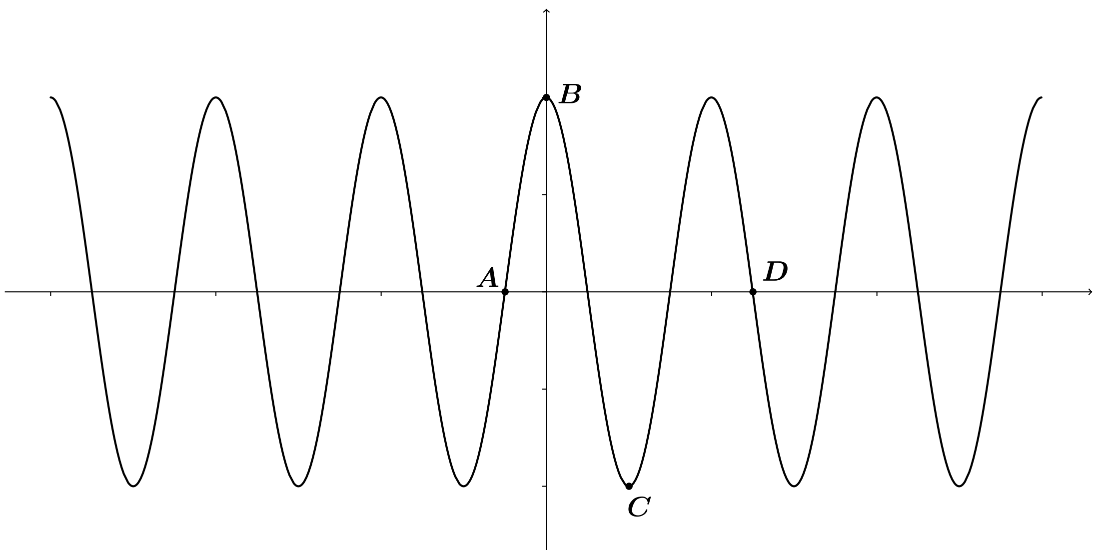
(d) The diagram below shows a graph of the function $f(x) = 10\cos 2x$, for $-2\pi \leq x \leq 2\pi$.

(i) Determine the coordinates of each of the points A, B, C and D, as marked in the diagram.

(ii) Write down the period of this function.

(iii) Write down the range of this function.

[5 marks]



Q5

(a) Differentiate the function $f(x) = 2x - x^2 + 5$ with respect to x from first principles.

[4 marks]

(b) Given that $y = x^2 + \dfrac{16}{x^2}$, find $\dfrac{dy}{dx}$.

Hence verify that $y = x^2 + \dfrac{16}{x^2}$ has a minimum value at $x = 2$.

[6 marks]

(c) Find the value of the integral

$$\int_0^1 e^{-2t}\, dt$$

correct to 3 decimal places.

[4 marks]

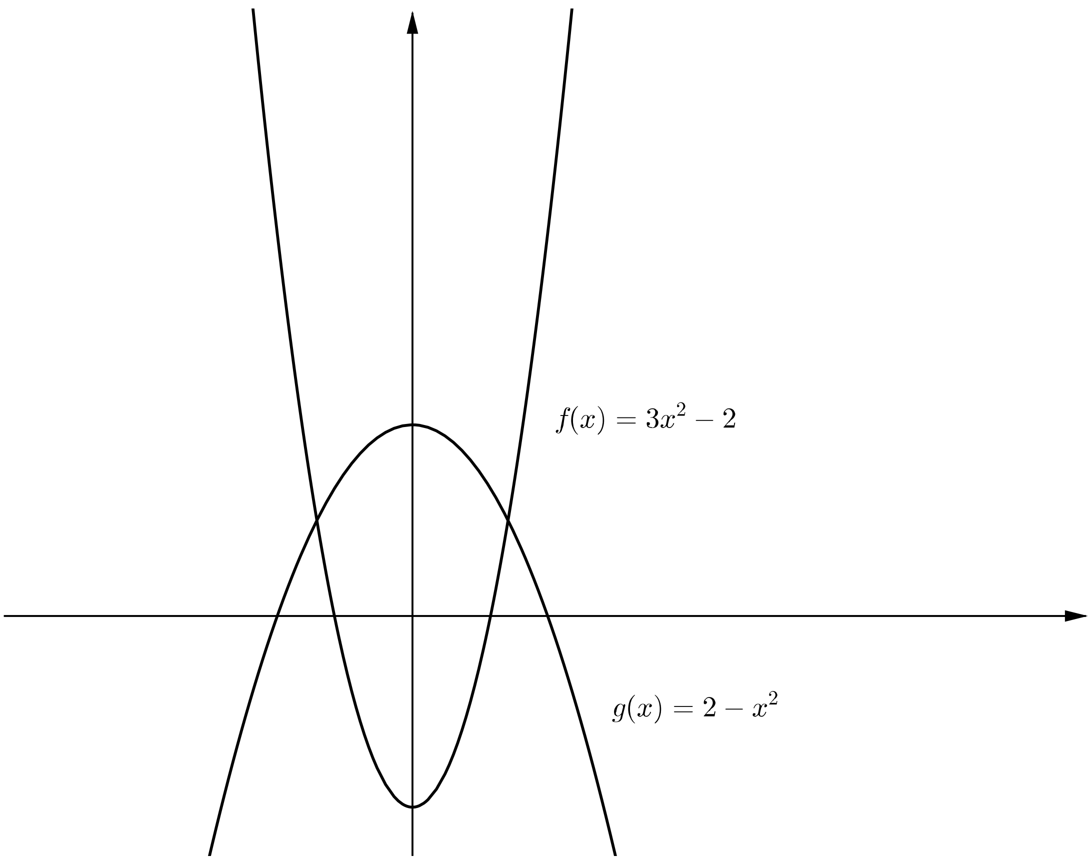
Question 5 continues overleaf

(d) The diagram below shows the graphs of the functions $f(x) = 3x^2 - 2$ and $g(x) = 2 - x^2$.

(i) Determine the coordinates of the two points at which the curves intersect.

(ii) Calculate the total area of the shaded region enclosed by the two curves.

[6 marks]



Cork Institute of Technology

CIT Mathematics Examination, 2015

Paper 2 — Sample Paper B

Answer **ALL FIVE** questions.

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Total marks available: 100 marks.

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[P.T.O.]

Q1

(a) (i) Write each of the following complex numbers in polar form:

- $z = 1 + \sqrt{3}i$
- $w = -2\sqrt{3} + 2i$

(ii) Plot each of the numbers z, w on an Argand diagram.

(iii) Evaluate zw, giving your answer in both cartesian and polar forms.

(iv) Evaluate $\frac{z}{w}$, giving your answer in both cartesian and polar forms.

(v) Use De Moivre's Theorem to find $\left(\frac{z}{w}\right)^6$, giving your answer in both cartesian and polar forms.

[11 marks]

(b) The nth term of a sequence is given by $u_n = 3(2^n)$ for $n \in \mathbb{N}$.

(i) Prove that this sequence is geometric.

(ii) Write down the first three terms of this sequence.

(iii) Find the sum of the first 10 terms.

[5 marks]

(c) Express the recurring decimal $0.366666\ldots$ in the form $\frac{p}{q}$ where $p, q \in \mathbb{N}$.

[4 marks]

Q2

(a) Solve the following simultaneous equations for x and y:

$$\begin{aligned} x^2 - 2y^2 &= 2 \\ 3x - y &= 7 \end{aligned}$$

[5 marks]

(b) Given that $x = -2$ is a root of the cubic equation $x^3 + tx^2 + 3x - 10 = 0$, find the value of t.

Also find all other roots of this equation.

[4 marks]

(c) Solve each of the following equations for x:

(i) $\log_2(3x + 2) = 5, \quad x \in \mathbb{R}, \quad x > -\dfrac{2}{3}$

(ii) $\log_{10} 25 + 2\log_{10} 2 = x, \quad x \in \mathbb{R}, \quad x > 0$

(iii) $\log_4(5x + 1) = 2\log_4 3 + 1, \quad x \in \mathbb{R}, \quad x > 0$

[6 marks]

(d) Solve the inequality $\dfrac{x+1}{x-1} < 2$, where $x \in \mathbb{R}$ and $x \neq 1$.

[5 marks]

Q3

(a) Two fair dice (one red, the other blue) are each rolled once. Find the probability of each of the following:

(i) a 4, 5 or 6 on the red die and a 1, 2, 3 or 4 on the blue die;

(ii) an odd number on the red die and an even number on the blue die;

(iii) the sum of the numbers obtained is 3.

[5 marks]

(b) The lifetime X of a certain battery is normally distributed with a mean μ of 6 years and a standard deviation σ of 0.5 years.

(i) Find $P(X \geq 7)$

(ii) Find $P(X \geq 7.25)$

(iii) Find $P(5.75 \leq X \leq 7.25)$

[6 marks]

(c) A box contains 150 light bulbs, 15 of which are defective. A sample of 3 light bulbs is randomly selected from the box.

(i) What is the probability that all three bulbs are defective?

(ii) Find the probability that none of the three bulbs is defective.

(iii) Find the probability that the first bulb is defective and that the second and third bulbs are not defective.

(iv) Find the probability that the sample contains exactly one defective bulb.

[9 marks]

Q4

(a) The lengths of the sides of a triangle are 21, 17 and 10. The smallest angle in the triangle is A.

(i) Show that $\cos A = \dfrac{15}{17}$.

(ii) Without evaluating A, find $\cos 2A$.

[5 marks]

(b) (i) Find the centre and radius of each of the following circles:

- $c_1: \quad x^2 + y^2 = 16$
- $c_2: \quad x^2 + y^2 - 6x - 8y + 24 = 0$

(ii) Sketch these two circles on the same set of axes.

(iii) Prove that these two circles touch externally.

[6 marks]

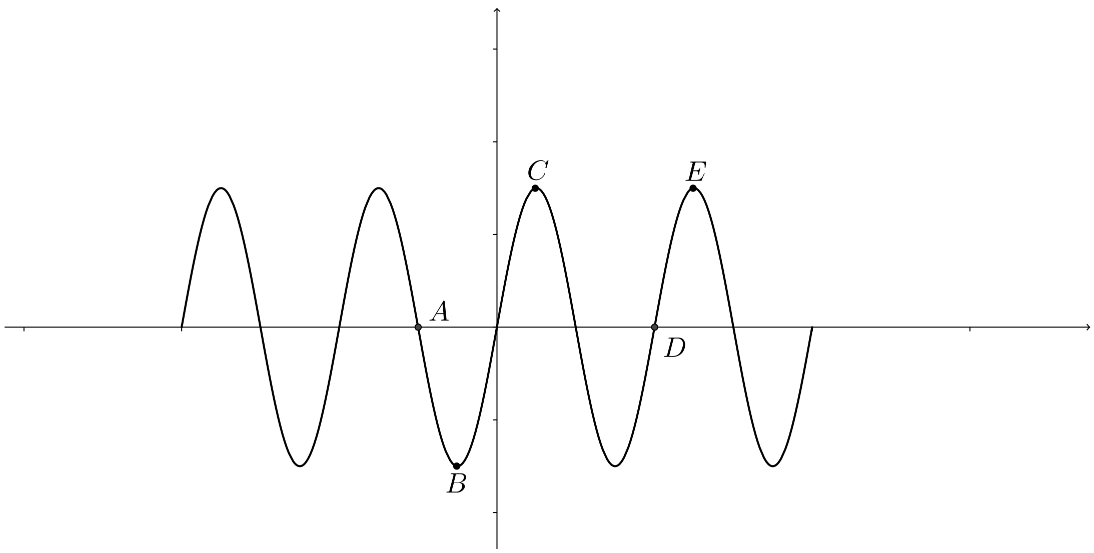
(c) The circumference of a circle is 30π cm. The area of a sector of this circle is 75π cm^2. Find the measure of the angle in this sector, giving your answer in both radians and degrees.

[5 marks]

Question 4 continues overleaf

(d) The diagram below shows a graph of the function $f(x) = 2\sin 4x$, for $-\pi \leq x \leq \pi$.
Determine the coordinates of each of the points A, B, C, D and E, as marked in the diagram.

[4 marks]



Q5

(a) Given that $y = x^2 e^{-2x}$, find $\dfrac{dy}{dx}$.
Hence verify that $y = x^2 e^{-2x}$ has a maximum value at $x = 1$.

[5 marks]

(b) The size $g(t)$ of a culture of bacteria after t days is given by

$$g(t) = Ae^{0.34t}$$

(i) You are told that the initial size of the culture is 4000.
Hence find the value of A.

(ii) Determine the size of the culture after 3 days.

(iii) After how many days will the population first exceed 20000?

(iv) Find the rate of change of the population after 2 days.

[6 marks]

(c) A function $f(x)$ has derivative $f'(x) = 2x + 2$. If $f(0) = 2$, then:

(i) Write down an expression for $f(x)$.

(ii) Find the average value of the function $f(x)$ over the interval $[1, 2]$.

[5 marks]

(d) Find

(i) $\displaystyle\int 2x^3 + 4\sin x \; dx$

(ii) $\displaystyle\int_1^4 \frac{1}{x^2} \; dx$

[4 marks]