Cork Institute of Technology

August Qualifying Mathematics Examination, 2009

Paper 1 (100 marks)

Saturday, 22 August 2009 — Morning

(Time: 2.5 Hours)

Answer **QUESTION 1** and **THREE** other questions.

Question 1 is worth 40 marks.
Questions 2–5 are worth 20 marks each.
Total marks available: 100 marks.

- The standard Mathematics Tables booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

NB This question is compulsory.

Answer any **eight** parts [5 marks each].

(a) (i) Kate starts a job at 08:15 hours and completes it at 12:45 hours. She is paid €9.50 per hour.
Find her total pay for the job.

(ii) The price of a camera has been reduced by 30% in a recent sale. It now costs €125.93.
What was the original price of the camera?

(b) (i) Maria is on holiday in New York and wants to buy a camera which costs \$267.35. If the current exchange rate is €1 = \$1.42, what is the price of the camera in euro, to the nearest cent?

(ii) Divide €500 in the ratio $2 : 3 : 5$.

(c) (i) An aeroplane has a cruising speed of 855 km per hour.
Express this cruising speed in metres per second.

(ii) The average centre-to-centre distance from Earth to the moon is 3.844×10^8 metres. Convert this distance to kilometres, giving your answer as a decimal number.

(d) Solve each of the following equations for x:

(i) $3(x + 4) = 5x + 6$

(ii) $x^2 - x - 6 = 0$

(e) (i) Write $\dfrac{16\left(\sqrt{2}\right)^3}{8}$ as a power of 2.

(ii) Find the value of $\dfrac{2a + 3b}{a - b}$ when $a = 5$ and $b = -3$.

(f) Let $f(x) = 2x - 1$, where $x \in \mathbb{R}$.

(i) Find the value of $f(-1)$;

(ii) Find the value of x for which $f(x) = 9$;

(iii) If $g(x) = \dfrac{9}{x}$, find $g(f(-1))$.

[Q1 continued overleaf]

(g) Let $z = 2 - i$ where $i^2 = -1$. Plot each of the following numbers on an Argand diagram:

(i) $z - 5i$

(ii) $-z$

(iii) $\bar{z} + iz$ (where $\bar{z}$ denotes the complex conjugate of z)

(h) The digits 0, 2, 4, 6, 8 are used to form four-digit codes. The first digit is never zero.

(i) How many different four-digit codes can be formed?

(ii) How many of these four-digit codes contain no repeated digits?

(i) A window is in the shape of a square surmounted by a semicircle. The semicircle has a radius of 30 cm.

Find the area of the window, correct to the nearest square centimetre.

Q2

(a) Solve each of the following equations for x:

(i) $\dfrac{6}{x-5} = \dfrac{2}{3} \qquad x \neq 5$

(ii) $x^2 - 2x - 5 = 0$

(iii) $3^{2x} = \dfrac{1}{81}$

[10 marks]

(b) Solve the following system of simultaneous equations for x and y:

$$\begin{aligned} 7x - 3y &= 13 \\ 3x + 4y &= -5 \end{aligned}$$

[5 marks]

(c) The admission prices for Blackrock Castle Observatory are as follows:

Adults: €6 each Children: €4 each

A school paid €190 for a recent visit to the observatory. Forty children went on the visit and the rest of the group consisted of adults.

How many adults were in the group?

[5 marks]

Q3

(a) Let $f(x) = x^2 + 3x - 4$, $x \in \mathbb{R}$.

(i) Find the value of $f(0)$;

(ii) Solve the equation $f(x) = 0$;

(iii) Find $f'(x)$, the derivative of $f(x)$.

(iv) Hence find the coordinates of the local minimum point of the curve $y = f(x)$.

(v) Use your answers to parts (i)–(iv) of this question to sketch the graph of $f : x \to x^2 + 3x - 4$, showing scaled and labelled axes.

[9 marks]

(b) Differentiate with respect to x:

(i) $(x^2 + 2x)(x^3 - 1)$

(ii) $(7x + 2)^4$

[6 marks]

(c) A train is travelling along a track. Suddenly, the brakes are applied. From the time the brakes are applied ($t = 0$ seconds), the distance travelled by the train, in metres, is given by

$$s = 120t - t^2$$

(i) What is the speed of the train at the moment the brakes are applied?

(ii) What is the speed of the train ten seconds after the brakes are applied?

(iii) How long does it take for the train to stop?

(iv) How far does the train travel in that time?

[5 marks]

Q4

(a) The line L_1 contains the points $p(-1, 7)$ and $q(3, -1)$.

(i) Find the slope of the line L_1.

(ii) Find the equation of the line L_1.

(iii) Hence determine the point at which L_1 intersects the x-axis and the point at which L_1 intersects the y-axis.

(iv) The line L_2 has equation $x - 2y + c = 0$ and passes through the point $(7, 5)$. Find the value of c.

(v) Prove that the lines L_1 and L_2 are perpendicular.

(vi) Find $L_1 \cap L_2$.

(vii) Draw a co-ordinate diagram on graph paper, showing the lines L_1 and L_2 and their point of intersection.

[12 marks]

(b) The circle C has equation $x^2 + y^2 = 10$.

(i) Verify that the point $(1, 3)$ lies on the circle C.

(ii) What is the centre of the circle C?

(iii) What is the radius of the circle C?

(iv) Verify that the point $(2, -3)$ lies outside the circle C.

(v) The line K has equation $x - 2y + 5 = 0$ and it intersects the circle at the points a and b. Find the co-ordinates of a and the co-ordinates of b.

[8 marks]

Q5

(a) A circle has centre o and radius 8 cm.
s and t are two points on the circle and $|\langle sot| = 30°$.

(i) Find the length of the shorter arc st, in centimetres, correct to one decimal place.

(ii) Find the area of the sector sot, in square centimetres, correct to two decimal places.

[5 marks]

[Q5 continued overleaf]

(b) A candle is in the shape of a cylinder surmounted by a cone.The cylinder has diameter 10 mm and the cone has slant height 13 mm.

(i) Find the vertical height of the cone.

(ii) Given that the height of the cylinder is 5 times the vertical height of the cone, find the volume of the candle, correct to the nearest cubic millimetre.

[5 marks]

(c) The diagram shows a vertical pole which stands on level ground.

A cable joins the top of a pole to a point on the ground which is 10 m from the base of the pole.

The cable makes an angle of 65° with the ground.

(i) Find the height of the pole, correct to one decimal place.

(ii) Find the length of the cable, correct to one decimal place.

[4 marks]

(d) One side of a triangle has length 8 cm and another has length 3 cm. The angle between these two sides measures 117°.

(i) Find the length of the third side, correct to one decimal place.

(ii) Find the measure of each of the two remaining angles, correct to the nearest degree.

[6 marks]

Q6

(a) A committee of 4 people is to be chosen from a group of 3 men and 4 women.

(i) How many different committees can be formed?

(ii) On how many of these committees is there an equal number of men and women?

(iii) How many of these committees contain more women than men?

[7 marks]

(b) The letters of the word IRELAND are arranged at random.

(i) How many different arrangements are possible?

(ii) How many of these arrangements begin with the letter I?

(iii) An arrangement is selected at random from all of the possible arrangements. Find the probability that it begins and ends with a vowel.

[4 marks]

(c) One hundred students, girls and boys, took an examination recently. The grades achieved are recorded in the following table:

Grade	**A**	**B**	**C**	**D**	**E**
Number of Girls	4	16	10	6	4
Number of Boys	8	20	18	10	4

If a student is selected at random, find the probability that the student is

(i) a girl;

(ii) a boy who got grade B;

(iii) a boy or girl who got grade A;

(iv) a girl who got grade C or D.

[5 marks]

(d) The table below shows the number of goals scored in a recent soccer tournament:

Number of goals scored	0	1	2	3	4
Number of matches	3	12	17	6	2

(i) Find the total number of goals scored in the tournament.

(ii) Find the mean number of goals scored in a match.

[4 marks]