

Cork Institute of Technology

Mathematics Qualifying Examination, 2009

Paper 2 (100 marks)

Saturday, 22 August 2009 — Afternoon

(Time: 2.5 Hours)

Answer **ANY FIVE** questions.

Each question is worth 20 marks.

Total marks available: 100 marks.

- The standard **Mathematics Tables** booklet is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.

[P.T.O.]

Q1

- (a) Solve the following system of simultaneous equations:

$$\begin{aligned}3x - 2y + z &= 4 \\x + y - 2z &= 5 \\4x - y + 3z &= -3\end{aligned}$$

[6 marks]

- (b) Show that $x = 3$ is a root of the cubic equation
 $x^3 - x^2 - x - 15 = 0$.

Find all other roots of this equation.

[6 marks]

- (c) Solve for x the equation

$$\log_5 x = 1 + \log_5 \left(\frac{5}{x} \right), \quad x \in \mathbb{R}, \quad x > 0$$

[4 marks]

- (d) Solve for x : $|4x + 5| < 20$.

[4 marks]

Q2

- (a) Three numbers are in arithmetic sequence. Their sum is 33 and their product is 792. Find the three numbers.

[4 marks]

- (b) In a geometric series, the second term is 8 and the fifth term is 27.

- (i) Find the first term and the common ratio.
(ii) Find, in terms of n , the sum of the first n terms.

[5 marks]

- (c) Express the recurring decimal $0.434343\dots$ in the form $\frac{p}{q}$
where $p, q \in \mathbb{N}$.

[5 marks]

- (d) Find the coefficient of x^3 in the binomial expansion of $\left(2x + \frac{1}{x}\right)^5$.

[6 marks]

Q3

- (a) (i) Given that $z = 2 - i$, calculate $|3z - 1|$ where $i^2 = -1$.
(ii) Express $-\sqrt{3} + i$ in the form $r(\cos \theta + i \sin \theta)$, where $i^2 = -1$.
(iii) Given that $w = 4 - 3i$, find the real number t such that $w^2 + tw$ is real.

[6 marks]

- (b) A is an acute angle such that $\tan A = \frac{8}{15}$.

Without evaluating A , find

- (i) $\cos A$
(ii) $\sin 2A$

[4 marks]

- (c) Find the two solutions of the equation

$$4 \sin^2 x - 3 \cos x - 3 = 0, \text{ where } 0^\circ \leq \theta \leq 180^\circ$$

Give your answers correct to the nearest degree.

[6 marks]

- (d) The area of a sector of a circle is 27 cm^2 . The length of the radius of this circle is 6 cm. Find, in radians, the measure of the angle in this sector.

[4 marks]

Q4

- (a) If $A = \begin{bmatrix} 3 & 5 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, find
(i) $A - B$
(ii) $A^{-1}B$

[5 marks]

[Q4 continued overleaf]

(b) rst is a triangle where

$$\vec{r} = -\vec{i} + 3\vec{j} \quad ; \quad \vec{s} = 4\vec{i} + 7\vec{j} \quad ; \quad \vec{t} = 6\vec{i} + 5\vec{j}$$

- (i) Express \vec{sr} , \vec{st} and \vec{tr} in terms of \vec{i} and \vec{j} .
- (ii) Find the length of each side of the triangle rst .
- (iii) Find the measure of the angle $\langle rst \rangle$.

[10 marks]

- (c) (i) Find a unit vector in the direction of $\vec{p} = 2\vec{i} + \vec{j}$.
- (ii) The vector $\vec{q} = 3\vec{i} + k\vec{j}$ (where $k \in \mathbb{R}$) is perpendicular to \vec{p} . Find the value of k .

[5 marks]

Q5

(a) Find:

- (i) the derivative of the function $f(t) = \sin^4 t$ with respect to t ;
- (ii) the value of the derivative of the function $y = x^2 \ln 2x$ at $x = \frac{1}{2}$.

[5 marks]

(b) The parametric equations of an ellipse are:

$$x = 2 \cos 2t \quad \text{and} \quad y = 3 \sin 2t \quad \text{where } 0 \leq t \leq \pi$$

Find the value of $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$.

[6 marks]

(c) Let $f(x) = xe^{-2x}$, $x \in \mathbb{R}$.

Show that $f(x)$ has a local maximum and find the coordinates of this local maximum point.

[5 marks]

(d) Find the slope of the tangent to the curve

$$2x^2 - 3y^2 = 15$$

at the point $(3, -1)$.

[4 marks]

Q6

(a) Find

(i)
$$\int 10x^4 + 4 \, dx$$

(ii)
$$\int \frac{1}{x^3} + \frac{1}{x} \, dx$$

[4 marks]

(b) Evaluate

(i)
$$\int_0^{\frac{\pi}{4}} \cos 4x \sin 2x \, dx$$

(ii)
$$\int_0^2 \frac{2x}{\sqrt{x^2 + 1}} \, dx$$

(iii)
$$\int_0^4 \frac{11}{x^2 + 16} \, dx$$

[10 marks](c) Calculate the area of the region enclosed by the curve $y = 9 - x^2$ and the line $y = x + 7$.**[6 marks]**